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# NAVAL POSTGRADUATE SCHOOL

Monterey, California



## THESIS

CERENKOV RADIATION: TIME DEPENDENT  
B FIELD OVER A FINITE PATH

by

Kathleen Marie Lyman

June 1986

Thesis Advisor:

John R. Neighbours

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Cerenkov Radiation: Time Dependent B Field  
Over a Finite Path

by

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June 1986



## ABSTRACT

This preliminary study investigated the magnetic field radiated from a passing charge bunch traveling over a finite path. Beginning with the infinite path case for a ramp front charge distribution, limits were derived to solve for the magnetic radiation field over a finite path. Radiation pulses were computed and graphed for many different positions of an observer with respect to the beam line. Comparisons of results show that the similarity in pulse shapes does not depend exclusively on the observer's position with respect to the Cerenkov region, but also on certain time conditions in each case.

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## I. INTRODUCTION

### A. HISTORY

Observations of a bluish-white light near a strong radioactive source had been recorded by workers before this phenomenon was understood. This was during a time (circa 1910) when the electromagnetic theory of light was well known and there was increased study in the area of optics and luminescence. The study of phosphorescence and fluorescence dominated, and the discovery of Cerenkov radiation was postponed due to the complexity of these forms of luminescence and the fact that Cerenkov radiation was weak in comparison. However, eventually the work on Cerenkov radiation developed, and was brought about through the study of phosphorescence and fluorescence. [Ref.1: p.1]

In 1926, Mallet took the first steps to study this phenomenon. He discovered that when a transparent material is placed near a strong radioactive source, the same bluish-white light would be emitted in a wide variety of cases. This light spectrum was continuous and did not contain the line spectrum characteristics of fluorescence. He also discovered that it differed in other respects from other forms of luminescence. The study of the phenomenon was not pursued again until 1934, when Cerenkov began a series of experiments which lasted until 1938. During this same time, Frank and Tamm proposed their theory (1937); there

was excellent correlation between this theory and Cerenkov's experimental results. [Ref.1: pp. 1-2]

Research in this area continued and with the development of the photomultiplier, the study in this area became more active. [Ref.1: p. 2]

## B. CERENKOV EFFECT

If a fast moving electron passes through a transparent medium, the atoms around the electron will become distorted and polarized. If the speed of the electron approaches that of light in the medium, then the polarization field is not symmetric. Symmetry is preserved in the azimuthal plane, "but along the axis there is a resultant dipole field which will be apparent even at large distances from the track of the electron." [Ref.1: p. 4] Because of this field, each element along the electron track will radiate a brief electromagnetic pulse. [Ref.1: p. 4]

Generally, these radiated wavelets from all parts of the track will interfere destructively and there will be no resultant field. However, if the particle velocity is higher than the phase velocity of light in the medium, the wavelets will be in phase and there will be a resultant field at a distant point of observation. This is observed only at a particular angle  $\theta$  with respect to the particle path. [Ref.1: p. 5]

Figure 1 [Ref.1: p. 5] illustrates the coherence of the

wavelets formed from points  $P_1$ ,  $P_2$ , and  $P_3$ . If  $\beta c_0$  is the particle velocity, where  $c_0$  is the speed of light in a vacuum and  $n$  is the index of refraction and  $\Delta\tau$  is time, then  $AB = (\beta c_0)(\Delta\tau)$ , the distance traveled by the particle, and  $AC = (c_0/n)(\Delta\tau)$ , the distance traveled by light. Thus, we can obtain  $\cos \theta = 1/\beta n$ , which is called the "Cerenkov relation". [Ref.1: p. 5]

There exists a threshold velocity, determined by the relation  $\beta_{\min} = (1/n)$ , and below this, no radiation is emitted. When radiation is emitted, it occurs in the visible and the near visible. [Ref.1: p. 5-6]

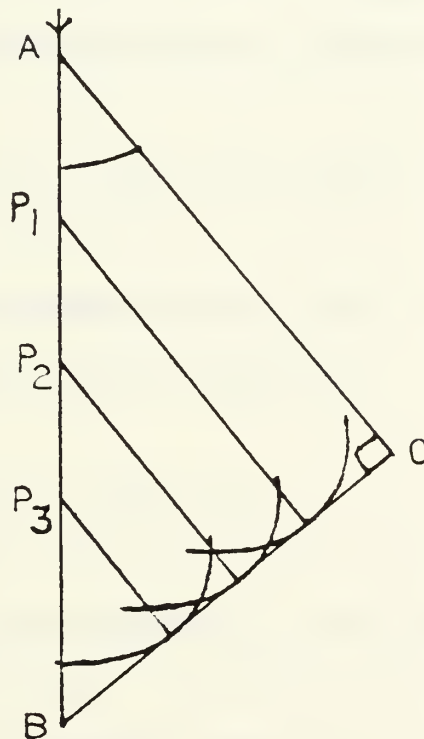


Figure 1. Huygens construction to illustrate coherence

Much of the research in Cerenkov radiation has been



limited to the optical regions, these being favored over the microwave region. The results from the optical radiation are expressed in terms of Fourier components for both the fields and the radiated power. [Ref.2: p..3750]

Since all the electrons in an accelerator bunch radiate coherently, microwave radiation can be important. The time structure of the fields formed by electron bunches that are radiated coherently was investigated by Professors Neighbours and Buskirk of the Naval Postgraduate School in their published paper of 1985. [Ref.2: p. 3750]

## II. THEORY AND OBJECTIVES

### A. MAGNETIC RADIATION FIELD

Neighbours and Buskirk proceeded by determining the potential from the moving charge distribution and then obtaining the B field (in cgs units) from the potential by

$$\vec{B} = \vec{\nabla} \times \vec{A} \quad (1)$$

The charge density function  $\rho$  and the current density  $j_v = \rho v / c_0$  ( $v$ , the velocity, is in the positive  $z$  direction) have been assumed and concentrated along the  $z$ -axis so that

$$\rho(\vec{r}, t) = \rho(z, t) \delta(x) \delta(y) \quad (2)$$

Since the charge is assumed to move with constant shape, the  $z$  and  $t$  dependence of the charge is

$$\rho(z, t) = \rho_0(z - vt) \quad (3)$$

where  $\rho_0$  and  $\rho$  are charge per unit length. [Ref.2: p. 3750]

The potential  $\vec{A}$  is found to be

$$\vec{A}(\vec{r}, t) = (\vec{v} / c_0) \int R^{-1} \rho(\vec{r}', t') dz \quad (4)$$

where  $\vec{R} = \vec{r} - \vec{r}'$  and  $t' = t - |\vec{r} - \vec{r}'|/c$  (the retarded time) and  $c$  is the speed of light in the medium. [Ref.2: p. 3750]

Equation (3) can be used in the potential equation and, defining a new variable  $u(z') = z' - vt'$ , with  $v$  defined as the particle velocity,  $\vec{A}$  can be written as

$$\vec{A}(\vec{r}, t) = (\vec{v}/c_0) \int R^{-1} \phi_0(u) dz \quad (5)$$

The function  $u(z')$  can be written as

$$u(z') = z' - vt + (v/c)[x^2 + y^2 + (z - z')^2]^{1/2} \quad (6)$$

since the motion of the charge is confined to the  $z$  axis. [Ref.2: pp. 3750-3751]

Because  $\vec{A}$  has only a  $z$  component, the  $B$  field, calculated from (1), has only  $x$  and  $y$  components; thus  $B_x = \partial(A_z)/\partial y$  and  $B_y = -\partial(A_z)/\partial x$ . Considering the  $x$  component only,

$$B_x = (v/c_0) \int (\partial R^{-1}/\partial y) \phi_0(u) dz' + (v/c_0) \int R^{-1} (\partial \phi_0(u)/\partial y) dz' \quad (7)$$

At large distances, the first integral can be neglected since it will fall off as  $R^{-2}$ . Then the  $x$ -component of the  $B$  field can be written as

$$B_x = (v^2/cc_0) \int (y/R^2) \phi_0'(u) dz' \quad (8)$$

where  $\phi'_0(u)$  is the derivative with respect to  $u$ . Similarly, the  $y$ -component can be written as

$$B_y = (v^2/cc_0) \int (-x/R^2) \phi'_0(u) dz \quad (9)$$

Combining these two components and using cylindrical coordinates,  $(s, \theta, z)$  where  $s = (x^2 + y^2)^{1/2}$ , the magnitude of total magnetic field,  $B$ , is written as

$$B = (v^2/cc_0) \int (s/R^2) \phi'_0(u) dz \quad (10)$$

and occurs in the direction of  $\theta$ , i.e. tangential. [Ref.2: p. 3751]

A similar derivation can be made in order to find the magnitude of the  $E$  field. It is also true that, in the Cerenkov case,  $E/B = c/c_0$ , which, for plane waves, is the usual relation between the electric and magnetic fields. [Ref.2: p. 3752]

## B. TIME DEVELOPMENT

Considering the function  $u(z')$ , as described in equation (6), we can determine that the first two terms of the equation are a straight line in the  $u-z'$  plane and the last term is a hyperbola which opens in the positive  $u$  direction and has asymptotic slopes of  $1 \pm (v/c)$ . The straight line part of this function has a unit slope and a time dependent

intercept. The combination of these two curves results in the curve  $u(z')$ . Figure 2 [Ref.2: p. 3751] indicates what this curve would look like for the Cerenkov case with  $v > c$ . The time indicated in Figure 2 increases from  $t_1$  to  $t_3$ . The curve will move downward with increasing time due to the negative second term of equation (6). [Ref.2: p. 3751]

The contribution to the B fields of equation (10) is due to changing currents (where  $\rho'_0$  is nonzero). A ramp front current pulse, for example, will have a derivative which is a constant square valued pulse whose magnitude is  $\rho'_m$ . This pulse is depicted in the right side of Figure 2. Only the positive pulse is considered. [Ref.2: p. 3751]

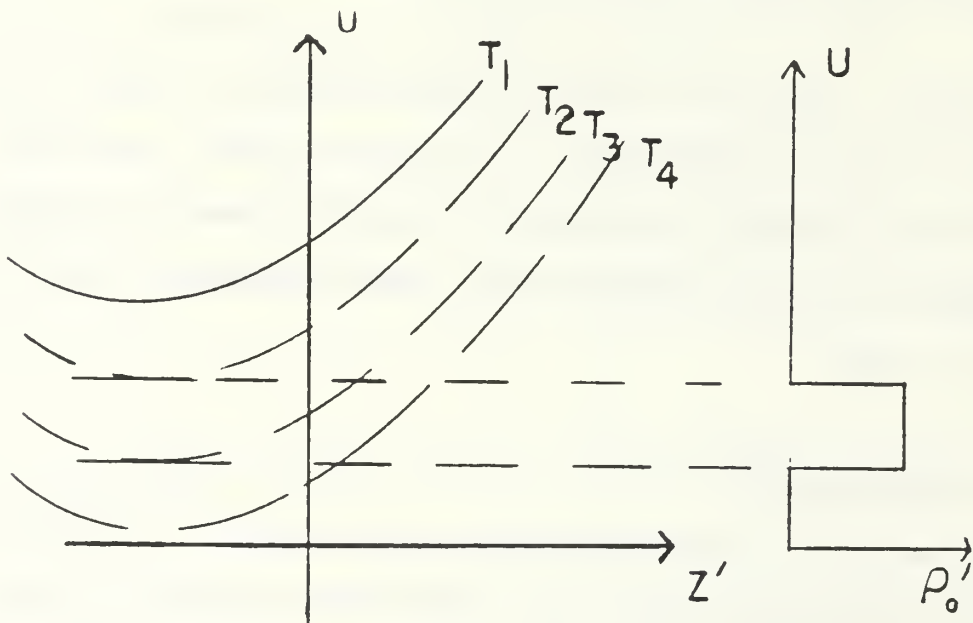


Figure 2. Function  $u = z - vt$

For this example, the function  $u(z')$  will be above the nonzero portion of  $\rho'_0$  for large negative times, and for



small positive times. During these periods,  $\phi'_0$  will be zero and therefore, the B field will be zero. When  $u(z')$  becomes tangent to the upper part of the  $\phi'_0$  pulse, namely  $u_1$ , then there is a nonzero contribution made by  $\phi'_0$  to the B field and it becomes nonzero. The magnitude of the B field will continue to increase until  $u(z')$  is tangent to the lower part of  $\phi'_0$ , called  $u_2$  in Figure 2. At later times the integral splits into two parts, and since  $\phi'_0$  is constant, the B field will decrease. This is due to the fact that the  $u$  function is turned upward and the area under the curve will decrease. [Ref.2: p. 3751]

### C. OBJECTIVES

The objective of this thesis is to solve for the magnitude of the B field over a finite path. In order to do this, the limits of integration for equation (10) must be found and a computer program written to solve the integral and graph the magnitude of the B field, for various situations. For this calculation, time begins at zero, when the beam is fired.

### III. EQUATION DEVELOPMENT

The derivation of equations in this chapter is based upon unpublished and untitled notes by Professor J.R. Neighbours.

#### A. CALCULATING THE B FIELD

Equation (10) can be written with finite limits of integration as

$$B = (v^2/cc_0) \int_{z'_i}^{z'_f} (sR^{-2}) \phi'_0(u) dz \quad (11)$$

If  $v^2/cc_0 = n\beta^2$  and  $\phi'_0(u) = \phi'_m = \text{constant}$ , then equation (11) becomes

$$B = n\beta^2 s \phi'_m \int_{z'_i}^{z'_f} R^{-2} dz \quad (12)$$

where  $s = (x^2 + y^2)^{1/2}$  and  $R^2 = s^2 + (z - z')^2$  or  $R^2 = s^2 + w^2$ , where  $w = z - z'$ . Substituting the variable  $w$  into equation (12) and integrating with respect to  $w$ , the solution becomes

$$B = n\beta^2 \phi'_m [\tan^{-1}(w_1/s) - \tan^{-1}(w_2/s)] \quad (13)$$

where  $w_1 = z - z'_i$  and  $w_2 = z - z'_f$ . Thus, the problem is to find the values of  $w_1$  and  $w_2$ .

## B. CALCULATING THE LIMITS OF INTEGRATION

### 1. Situations Encountered For The Finite Path

There are three basic situations encountered in this study of the B field and each depends upon the position of the observer in relation to the Cerenkov angle,  $\theta_c$ . Figures 3, 4, and 5 illustrate the three different situations. In each of these figures, the beam length is L, and the point, P(z,s), is the position of the observer.

The three situations can be related to the position of the minimum of the function  $u(z')$ , i.e., the path can be to the right, left, or centered about the minimum. If  $v/c=\beta'$ , and substituting the value of s, equation (6) can be written as

$$u(z') = z' - vt - \beta' [s^2 + (z-z')^2]^{1/2} \quad (14)$$

This function has a minimum at  $\tan \theta_c = \pm s/(z-z')$ ; therefore, the minimum occurs on the Cerenkov cone when  $z-z'$  is such that  $\theta = \theta_c$ .

The criterion for the path to be to the right is that  $\theta_1$  in Figure 3 must be greater than  $\theta_c$ ; for the path to be to the left,  $\theta_2$  in Figure 4 must be less than  $\theta_c$ ; for the path to be centered about the minimum,  $\theta_1$  must be less than  $\theta_c$  and  $\theta_2$  must be greater than or equal to  $\theta_c$ , as shown in Figure 5.

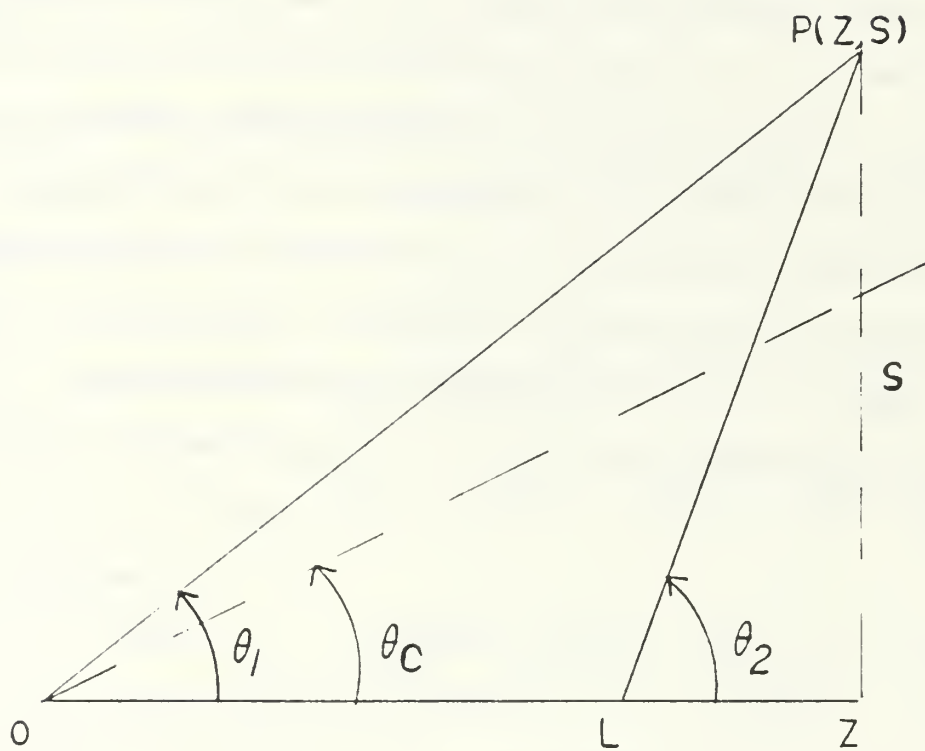


Figure 3. Path To The Right

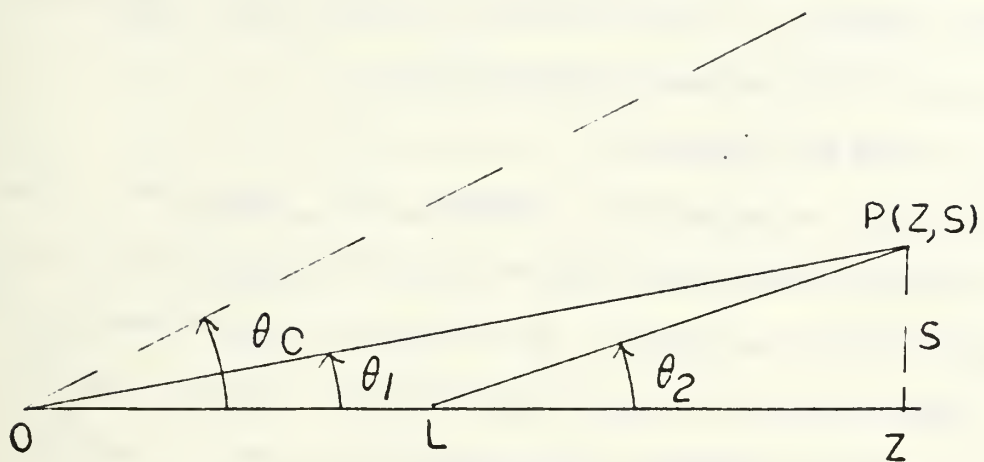


Figure 4. Path To The Left

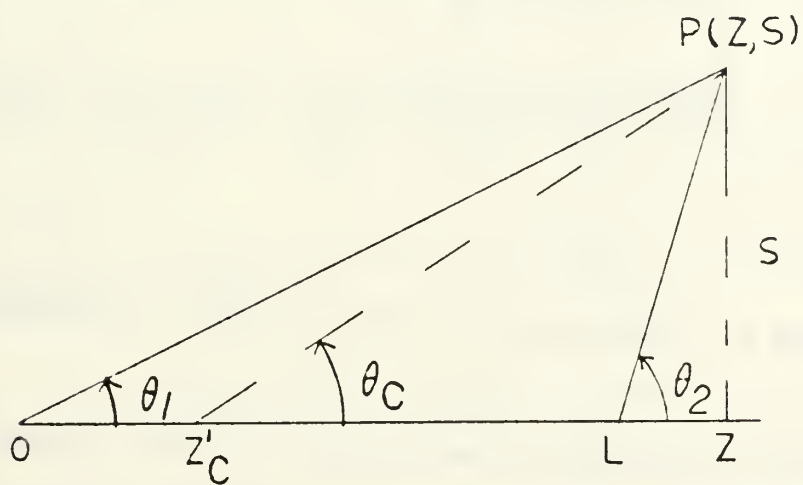


Figure 5. Path Centered About The Minimum



The rectangle in the  $u-z'$  plane (Figure 6) is formed by the path length,  $L$ , and the limits on the ramp function,  $u_1$  and  $u_2$ ; the corners are labeled ABCD.

a. Path To The Right

The relationship between the path and the minimum of  $u(z')$  is shown in Figure 6. The B field will be encountered first at time  $T_A$  and will go to zero again at time  $T_D$ . The limits of integration for the curve, as it passes through the rectangle bounded by  $T_A$ ,  $T_B$ ,  $T_C$ , and  $T_D$ , are the points of intersection of the curve and the rectangle.

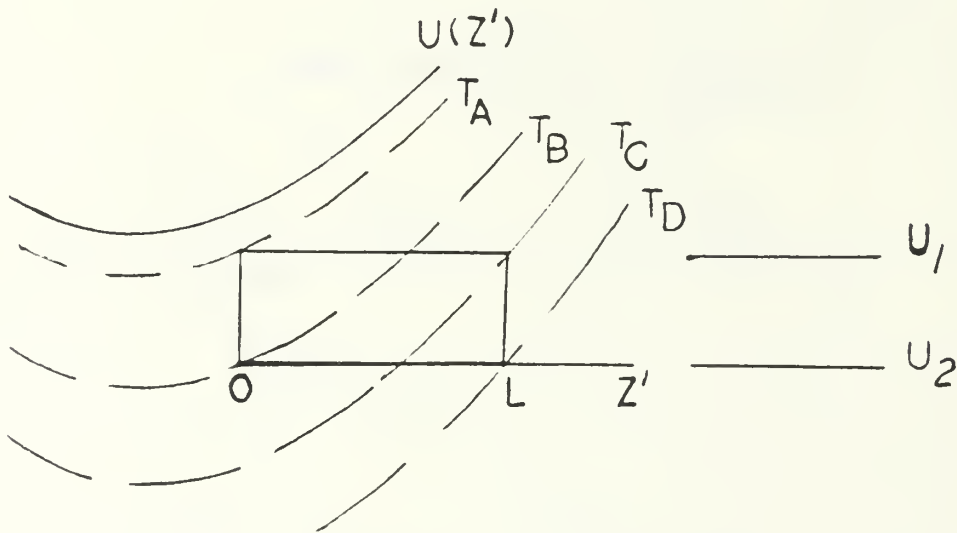


Figure 6. Relation of  $u(z')$  to Path to the Right

For the path to the right, as well as the other two cases, the values of these boundary times are found by solving equation (14), having substituted the appropriate values of  $z'$  and  $u(z')$ . The results are

$$T_A = [\beta'(s^2+z^2)^{1/2} - u_1]/v \quad (15)$$

$$T_B = [\beta'(s^2+z^2)^{1/2} - u_2]/v \quad (16)$$

$$T_C = [L + \beta'(s^2+(z-L)^2)^{1/2} - u_1]/v \quad (17)$$

$$T_D = [L + \beta'(s^2+(z-L)^2)^{1/2} - u_2]/v \quad (18)$$

The case illustrated in Figure 6 is one in which  $T_C$  is greater than  $T_B$ . The condition for this case to occur is that  $u_1 - u_2 < L + \beta'[(s^2+(z-L)^2)^{1/2} - (s^2+z^2)^{1/2}]$ . There are two other cases, namely, when  $T_C$  is less than  $T_B$  and  $T_C$  is equal to  $T_B$ . The derivation of the equations for the limits of integration for the first case will be described; the equations for the last two cases are found in a similar manner.

Suppose  $T_A < T < T_B$  and  $u(z)$  is positioned at time  $T$  as illustrated in Figure 7a. The lower limit of integration for equation (12),  $z'_i$ , is zero in this case and

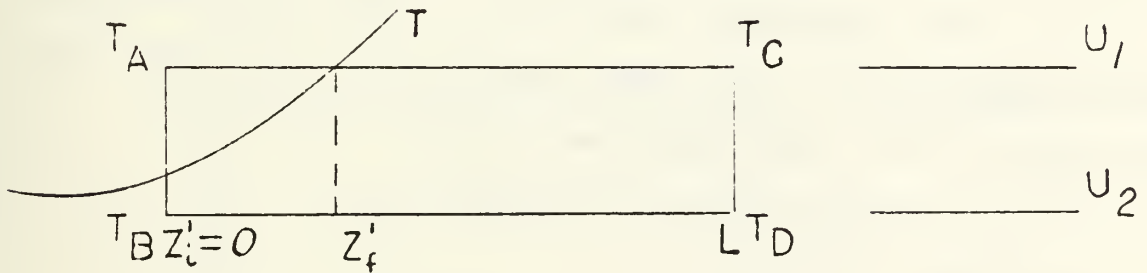


Figure 7a.  $T_C > T_B$ :  $T_A < T < T_B$

$z'_f$  is found by solving equation (14). If  $z' = z'_f$ ,  $u(z') = u_1$ , and a new variable,  $A_1 = u_1 + vt$ , is introduced, the solution of this quadratic equation becomes

$$z'_f = \{ (\beta'^2 z - A_1) + \beta' [(z - A_1)^2 - s^2 (\beta'^2 - 1)]^{1/2} \} / (\beta'^2 - 1) \quad (19)$$

Only the positive solution for the quadratic equation is valid because of the position of the minimum for the path to the right.

The last two situations for the  $T_c > T_b$  case are  $T_b < T < T_c$  and  $T_c < T < T_d$ , which are illustrated in Figures 7b and 7c, respectively. For these last two situations, a new variable,  $A_2$ , is used and defined as  $u_2 + vt$ . After solving equation (14) for the lower and upper integral limits, we find that for  $T_b < T < T_c$

$$z'_i = \{ (\beta'^2 z - A_2) + \beta' [(z - A_2)^2 - s^2 (\beta'^2 - 1)]^{1/2} \} / (\beta'^2 - 1) \quad (20)$$

and  $z'_f$  is calculated by using equation (19). For  $T_c < T < T_d$ ,  $z'_i$  is found by using equation (20) and  $z'_f = L$ .

As mentioned previously, there are two more cases for the path to the right, where  $T_c < T_b$  and  $T_c = T_b$ . For the case of  $T_c < T_b$ , the condition is that

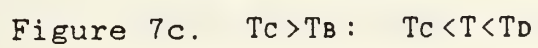
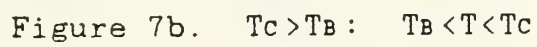
$$u_1 - u_2 > L + \beta' [(s^2 + (z - L)^2)^{1/2} - (s^2 + z^2)^{1/2}]$$

and for  $T_c = T_b$

$$u_1 - u_2 = L + \beta' [(s^2 + (z - L)^2)^{1/2} - (s^2 + z^2)^{1/2}]$$

Figures 8 and 9 illustrate the last two cases for the path to the right.

The limits of integration for these last cases are found in the same way as for the first case. A summary



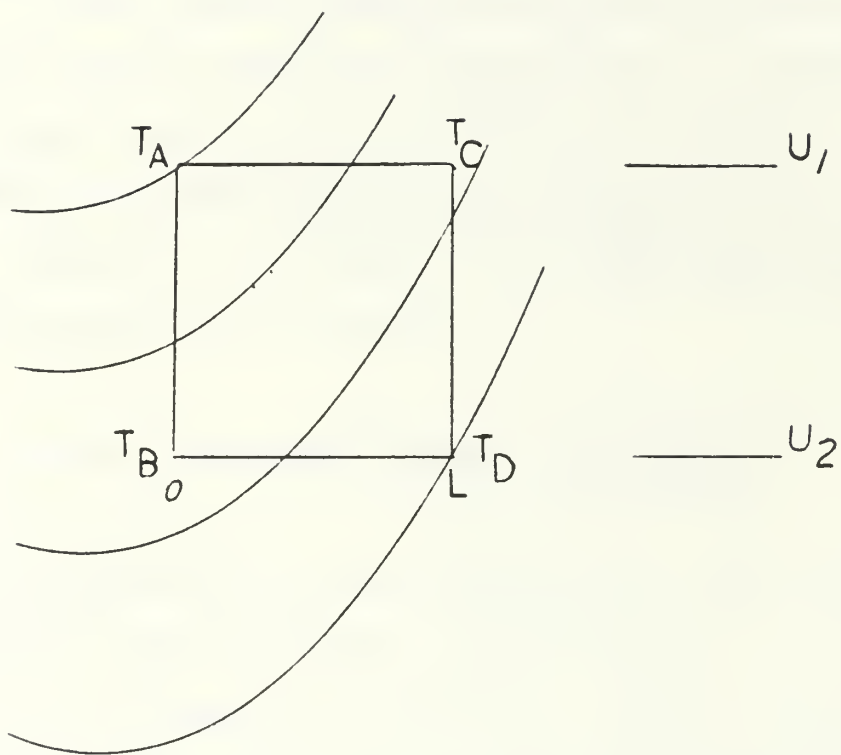


Figure 8. Path To The Right:  $T_C < T_B$

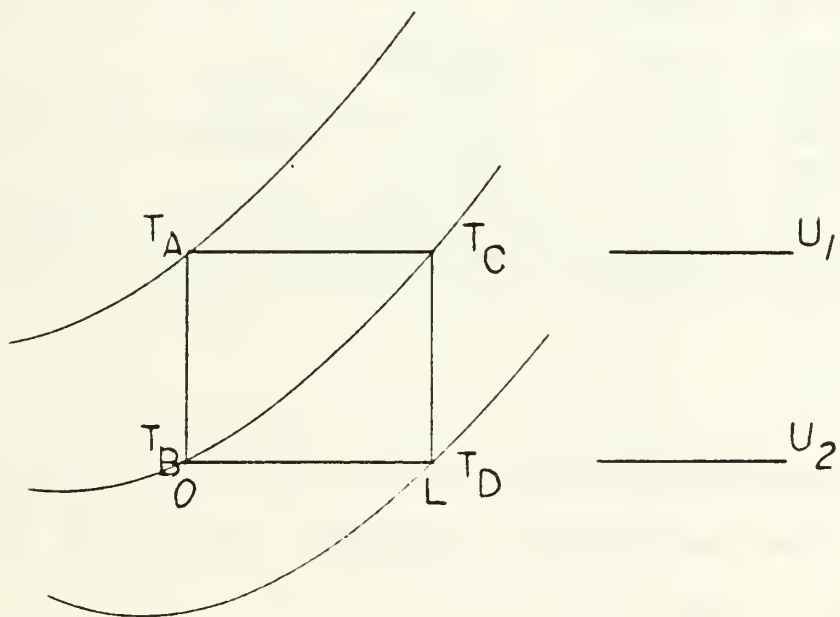


Figure 9. Path To The Right:  $T_c = T_b$



of the limits of integration for the path to the right is listed in Table 1, where

$$a = \{(\beta'^2 - A_2) + \beta'[(z - A_1)^2 - s^2(\beta'^2 - 1)]^{1/2}\} / (\beta'^2 - 1)$$

and

$$b = \{(\beta'^2 z - A_2) + \beta'[(z - A_2)^2 - s^2(\beta'^2 - 1)]^{1/2}\} / (\beta'^2 - 1).$$

TABLE 1

LIMITS OF INTEGRATION: PATH TO THE RIGHT

$T_C > T_B :$	$T_A < T < T_B$	$T_B < T < T_C$	$T_C < T < T_D$
$z'_i$	0	b	b
$z'_f$	a	a	L
$T_C < T_B :$	$T_A < T < T_C$	$T_C < T < T_B$	$T_B < T < T_D$
$z'_i$	0	0	b
$z'_f$	a	L	L
$T_B = T_C :$	$T_A < T < T_B$	$T_B < T < T_D$	
$z'_i$	0	b	
$z'_f$	a	L	

b. Path To The Left

The relationship between the minimum of  $u(z)$  and the path is shown in Figure 10.

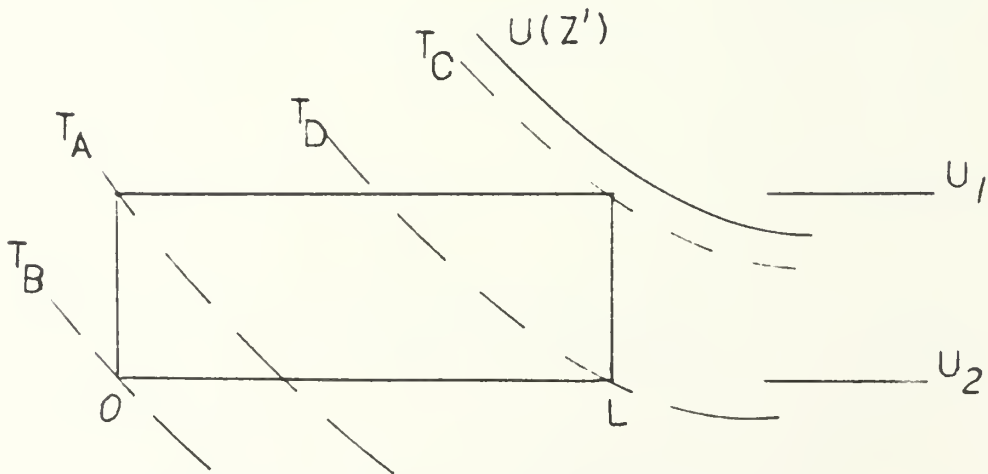


Figure 10. Relation of  $u(z')$  to Path to the Left

The limits of integration for the path to the left are found in the same manner as in the path to the right. However, due to the position of the minimum with respect to the path, only the negative part of the quadratic solution will be used. The three cases for the path to the left are summarized in Table 2; the conditions for each are included.

TABLE 2

CASES AND CONDITIONS FOR THE PATH TO THE LEFT

Cases	Conditions
$T_A > T_D$	$u_1 - u_2 < \beta' [(s^2 + z^2)^{1/2} - (s^2 + (z-L)^2)^{1/2}] - L$
$T_A < T_D$	$u_1 - u_2 > \beta' [(s^2 + z^2)^{1/2} - (s^2 + (z-L)^2)^{1/2}] - L$
$T_A = T_D$	$u_1 - u_2 = \beta' [(s^2 + z^2)^{1/2} - (s^2 + (z-L)^2)^{1/2}] - L$

These cases are shown in Figures 11a, b, and c; the limits of integration are listed in Table 3 with

$$aa = \{(\beta'^2 z - A_1) - \beta'[(z - A_1)^2 - s^2(\beta'^2 - 1)]\}/(\beta'^2 - 1)$$

and

$$bb = \{(\beta'^2 z - A_2) - \beta'[(z - A_2)^2 - s^2(\beta'^2 - 1)]\}/(\beta'^2 - 1)$$

TABLE 3

LIMITS OF INTEGRATION: PATH TO THE LEFT

$T_A < T_D$ :	$T_c < T < T_A$	$T_A < T < T_D$	$T_D < T < T_B$
$z_i$	aa	0	0
$z_f$	L	L	bb
$T_A > T_D$ :	$T_c < T < T_D$	$T_D < T < T_A$	$T_A < T < T_B$
$z_i$	aa	aa	0
$z_f$	L	bb	bb
$T_A = T_D$ :	$T_c < T < T_A$	$T_D < T < T_B$	
$z_i$	aa	0	
$z_f$	L	bb	

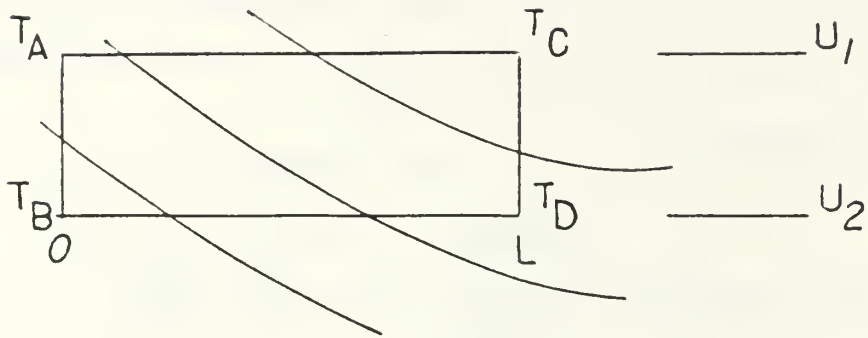


Figure 11a. Path To The Left:  $T_A > T_D$

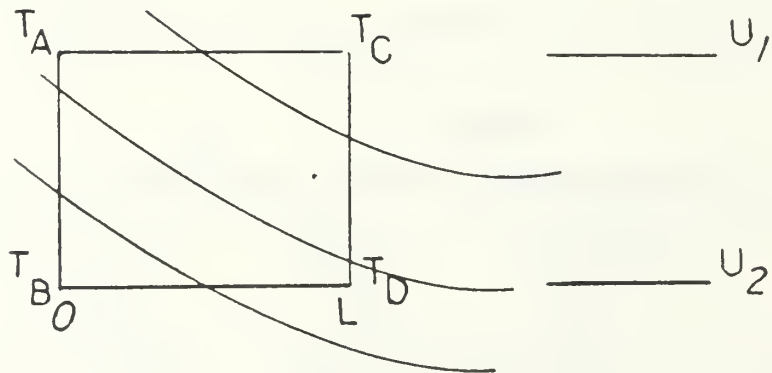


Figure 11b. Path To The Left:  $T_A < T_D$

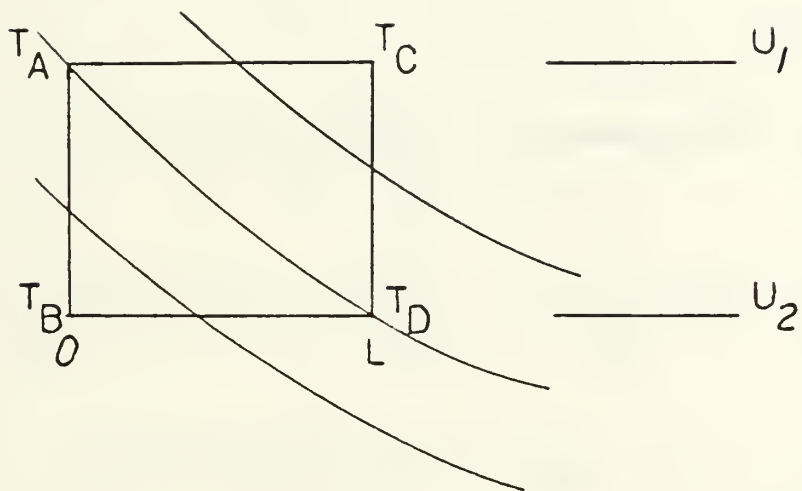


Figure 11c. Path To The Left:  $T_A = T_D$

c. Centered About The Minimum

This case is slightly different from the first two, as can be seen in Figure 12. The minimum occurs between 0 and L at  $z'_c$ , so that as shown in Figure 5, the position of the observer makes the Cerenkov angle with respect to the direction of the beam. For this case, new

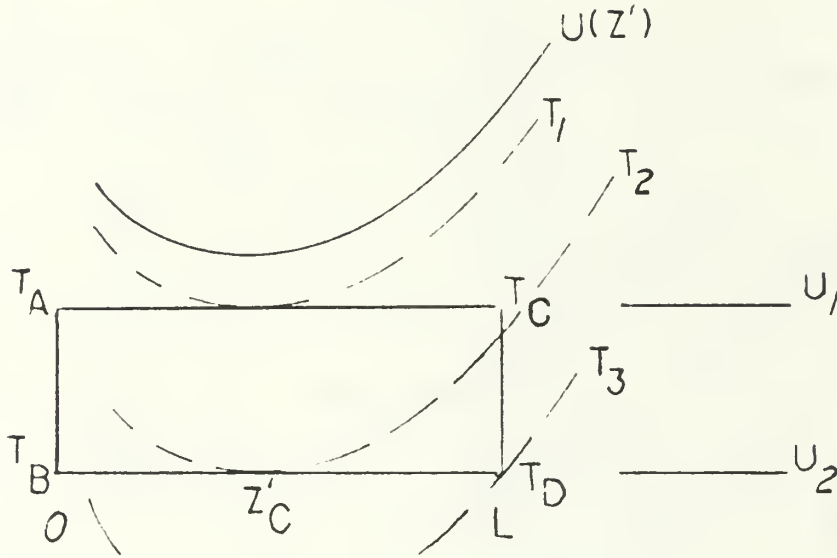


Figure 12. Centered About the Minimum

times must be introduced:  $T_1$ , when the minimum just contacts the  $u_1$  line,  $T_2$ , when the minimum contacts the  $u_2$  line and  $T_3$ , the final time. Calculating the times from equation (14), we find that the results are

$$T_1 = \{z'_c + \beta'[s^2 + (z - z'_c)^2]^{1/2} - u_1\}/v$$

$$T_2 = \{z'_c + \beta'[s^2 + (z - z'_c)^2]^{1/2} - u_2\}/v$$

and  $T_3$  will be the larger of the  $T_B$  or  $T_D$ , as previously defined in equations (16) and (18), respectively.

Considering the situation where  $T_1 < T < T_2$ , as shown in Figure 13, it can be seen that both solutions to the quadratic equation may be utilized, such that

$$z'_+ = \{ (\beta'^2 z - A_1) + \beta' [(z - A_1)^2 - s^2 (\beta'^2 - 1)]^{1/2} \} / (\beta'^2 - 1) \quad (21)$$

and

$$z'_- = \{ (\beta'^2 z - A_1) - \beta' [(z - A_1)^2 - s^2 (\beta'^2 - 1)]^{1/2} \} / (\beta'^2 - 1) \quad (22)$$

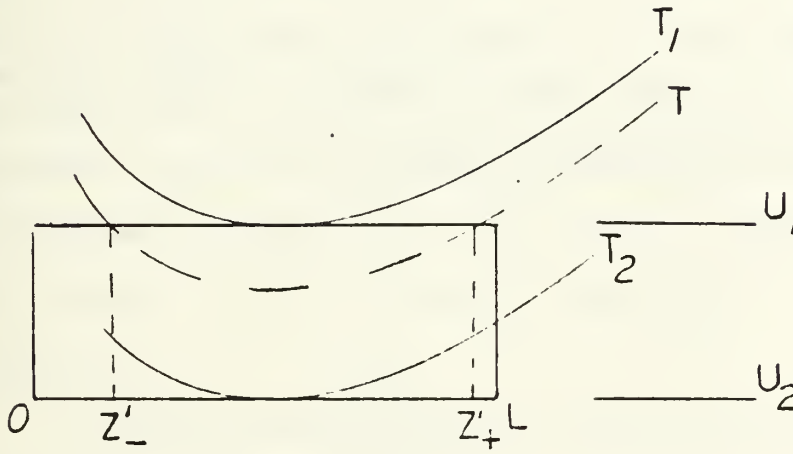


Figure 13.  $T_1 < T < T_2$

With these two solutions, the following conditions, listed in Table 4, are imposed in order to choose the proper limits of integration.

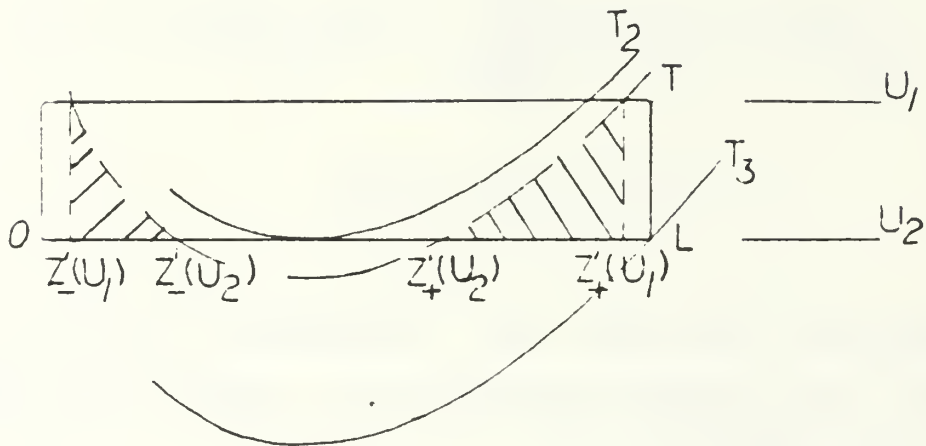


TABLE 4

CONDITIONS FOR THE LIMITS OF INTEGRATION:  $T_1 < T < T_2$ 

Condition	$z'_i$	$z'_f$
$z'_- \leq 0$	0	
$z'_- > 0$	$z'_-$	
$z'_+ \geq L$		L
$z'_+ < L$		$z'_+$

For the last case,  $T_2 < T < T_3$ , there are two integrals to solve, as shown in Figure 14.

Figure 14.  $T_2 < T < T_3$

For the first integral,  $z'_-(u_1)$  is found by using equation (22) and

$$z'_+(u_2) = \{\beta'^2 z - A_2\} - \beta' [(z - A_1)^2 - s^2 (\beta'^2 - 1)]^{1/2} / (\beta'^2 - 1) \quad (23)$$

In this case  $z'_f$  will always be  $z'_-(u_2)$ . However,  $z'_i$  will equal  $z'_-(u_1)$  when the latter value is greater than zero; otherwise,  $z'_i$  will equal zero.

Similarly, the limits for the second integral,  $z'_+(u_1)$  and  $z'_+(u_2)$  will be the corresponding solutions to the above equations in which the second term is positive. In this case,  $z'_i$  will always be  $z'_+(u_2)$ , whereas  $z'_f$  will equal  $z'_+(u_1)$  if the latter is less than L, otherwise  $z'_f$  will equal L.

The total B field will be the sum of these two integrals.

Table 5 lists a summary of the limits of integration for the path centered about the minimum.

TABLE 5

LIMITS OF INTEGRATION: PATH CENTERED ABOUT THE MINIMUM

	$z'_- \leq 0$	$z'_- > 0$	$z'_+ \geq L$	$z'_+ < L$
$T_1 < T < T_2$				
$z'_i$	0	$z'_-$		
$z'_f$			L	$z'_+$
$T_2 < T < T_3$				
FIRST INTEGRAL:				
$z'_i$	0	$z'_-$		
$z'_f = z'_-(u_2)$				
SECOND INTEGRAL:				
$z'_i = z'_+(u_2)$				
$z'_f$			L	$z'_+(u_1)$

### C. SUMMARY

In each of the above situations, if the values for  $z_i$  and  $z_f$  are substituted into the equations for  $w_1$  and  $w_2$ , respectively, the limits of integration can be calculated and used to solve equation (13), the magnitude of the B field.

#### IV. CALCULATIONS AND ANALYSIS

##### A. CALCULATIONS

Having derived the formulae for the limits of integration, the next step was to write a computer program to calculate the B fields and graph it against time.

The FORTRAN program is interactive and has a variable input for the values of  $u_1$ ,  $u_2$ ,  $\beta$ ,  $n$ ,  $f'_m$ ,  $s$ ,  $z$  and  $L$ . The values for the first five quantities were chosen to be:  $u_1=100$  cm,  $u_2=50$  cm,  $\beta=.99$ ,  $n=1.111111$  and  $f'_m=1$ . Various values for  $s$ ,  $z$  and  $L$  were used.

Based on the input, this program will choose what type of situation is occurring, whether right, left or center. It will calculate the B field during the time the  $u(z')$  curve transits the rectangle in the  $u-z'$  plane, and will graph B vs time(nsec).

The graphics part of this program is based on a code written by Professor J.R. Neighbours for use on a Textronix computer; the main part of the program is an original work.

Calculations were carried out for various beam lengths, using different values for  $s$ ; numerous values of  $z$  were used and the corresponding B field graphs were obtained. After studying these curves, a beam length of 1500 was chosen with  $s$  values of 1500 and 3000. Graphs for selected

values of  $z$  were compared; these graphs are shown in Figures 20 through 42, found in Appendix A.

## B. MINIMUM TIME

In analyzing the three situations, the minimum time taken for the radiation to reach the observer was determined. Figure 15 shows the situation which will be used to find the time in each of the three cases.

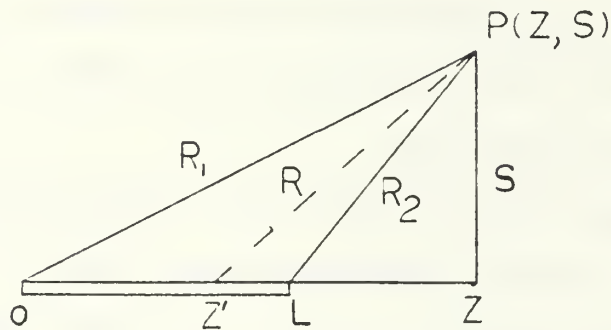


Figure 15. Path of Beam and Radiation ( $z$  and  $R$ )

The beam emerges at  $O$  and travels to  $L$  at constant velocity  $v$ , where  $v = \beta c$ . Radiation is emitted when the head of the beam is at  $z'$ . The radiation then travels at a constant velocity,  $c$ , to the observer at  $P$ .

The time it will take the radiation to reach the observer is given by

$$t(z') = z'/(\beta c) + (nR)/c \quad (24)$$

Since  $R^2 = (z - z')^2$ , equation 24 can be written as

$$t(z') = z'/\beta c_0 + n/c_0 [(z-z')^2 + s^2]^{1/2} \quad (24a)$$

As can be seen in Figure 16, there is a minimum time for the path centered about the minimum. It occurs at the point  $z'_c$  (at the Cerenkov angle,  $\theta_c$ ). The values used in this case are  $z=4000$ ,  $L=1500$  and  $s=1500$ . This graph verifies the following calculation for the minimum time occurring at the Cerenkov angle.

$$\begin{aligned} dt/dz' &= 1/\beta c_0 + (1/2)(n/c_0)[(z-z')^2 + s^2]^{-1/2} 2(z-z') \\ &= 1/\beta c_0 + (n/c_0)[(z-z')/R] \\ &= 1/\beta c_0 + (n/c_0)\cos \theta = 0 \end{aligned}$$

This equation will equal zero when the value of  $\cos \theta$  is  $1/n\beta$ ; this implies that  $\theta = \theta_c$ .

Figures 17 and 18 show that there is not minimum time over the length of the beam,  $L=1500$ , for the paths to the right and left.

Although this analysis confirms that the minimum occurs at the Cerenkov angle, there is no correlation between the time in Figures 17, 18 and 19, and the time in the B field radiation graphs. The problem stems from the fact that  $T_a$  through  $T_d$  depend on the values of  $u_1$  and  $u_2$ , and  $T_c$  and  $T_d$  also depend on  $L$ , while equation (24) depends on neither of these quantities.



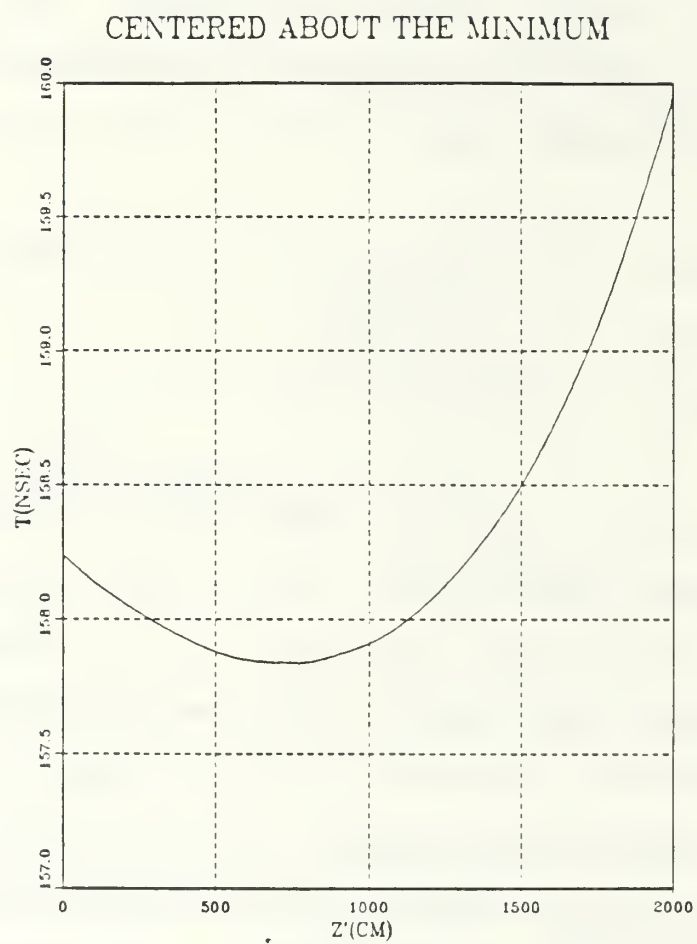


Figure 16: Centered About the Minimum

### PATH TO THE RIGHT

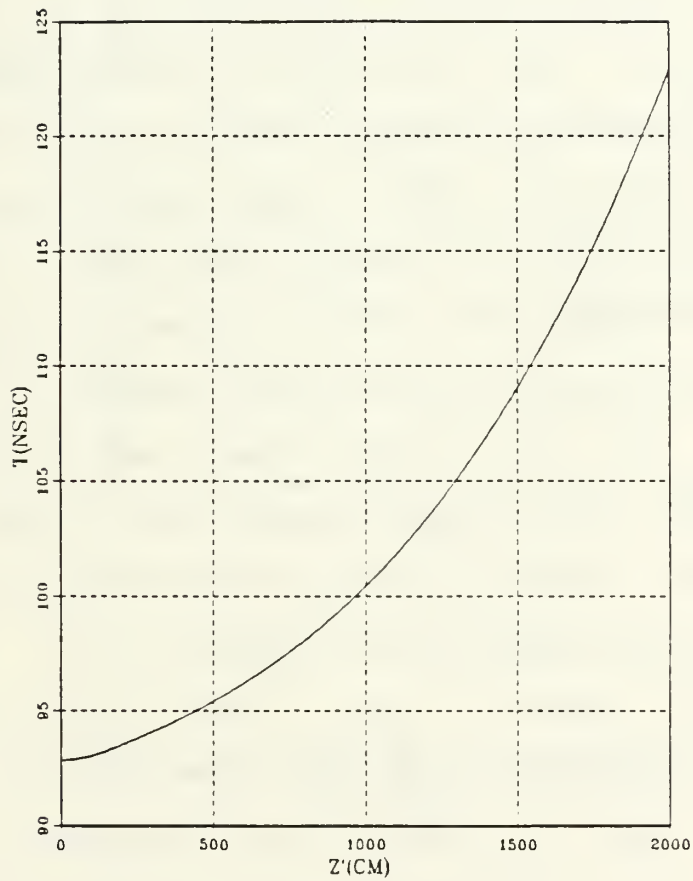


Figure 17: Path to the Right: No Minimum Time

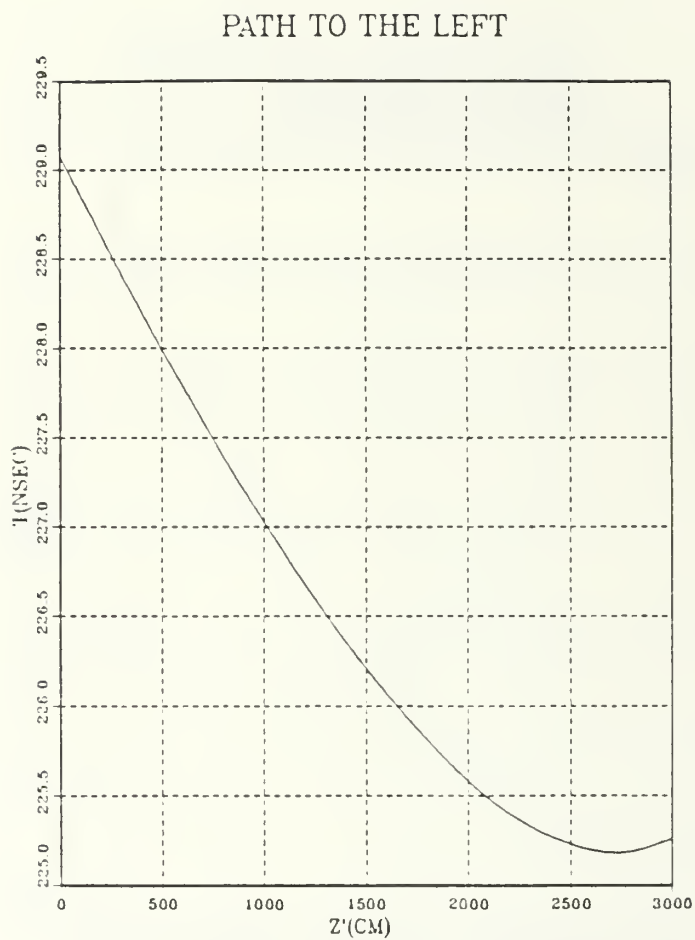


Figure 18: Path to the Left: No Minimum Time

### C. ANALYSIS OF THE B FIELD GRAPHS

In analyzing the B field graphs, it was decided to look for similar characteristics and shapes. In this analysis two cases were considered, each with  $L=1500$ ; the comparison was made between the graphs for  $s=3000$  and  $s=1500$ .

Figure 19 is a graph of  $s/L$  vs  $z/L$ . This graph indicates the boundary regions for time, i.e., line #1 is  $T_c=T_b$ , and line #2 is  $T_a=T_d$ ; these values are based on a  $\Delta u$  of 50 cm and the ratio  $\Delta u/L = .03333$ , where  $\Delta u = u_1 - u_2$ . The time region to the left of line #1 is  $T_c > T_b$ , and the region to the right of line #2 is  $T_a > T_d$ . The region between line #1 and #2 contains the times  $T_c < T_b$  and  $T_a < T_d$ . The graph also indicates the Cerenkov region, the area between the dashed lines. The regions labeled RIGHT, CENTER and LEFT correspond to the position of the observer as previously described. The boundary lines between these regions are the dashed lines. Each of the B fields graphs can be placed in different time regions as indicated in Figure 19. Figures 20 through 28 are all graphs of the path to the right for which  $T_c > T_b$ ; these graphs lie to the left of line #1 and are indicated by the symbol x in Figure 19. Figures 20 through 26 are for  $s=1500$ , while Figures 27 and 28 are for  $s=3000$ . Comparing these graphs, it can be seen that there is a similarity in shape; as the  $z$  value gets closer to the Cerenkov region, for each  $s$  case, the initial peak

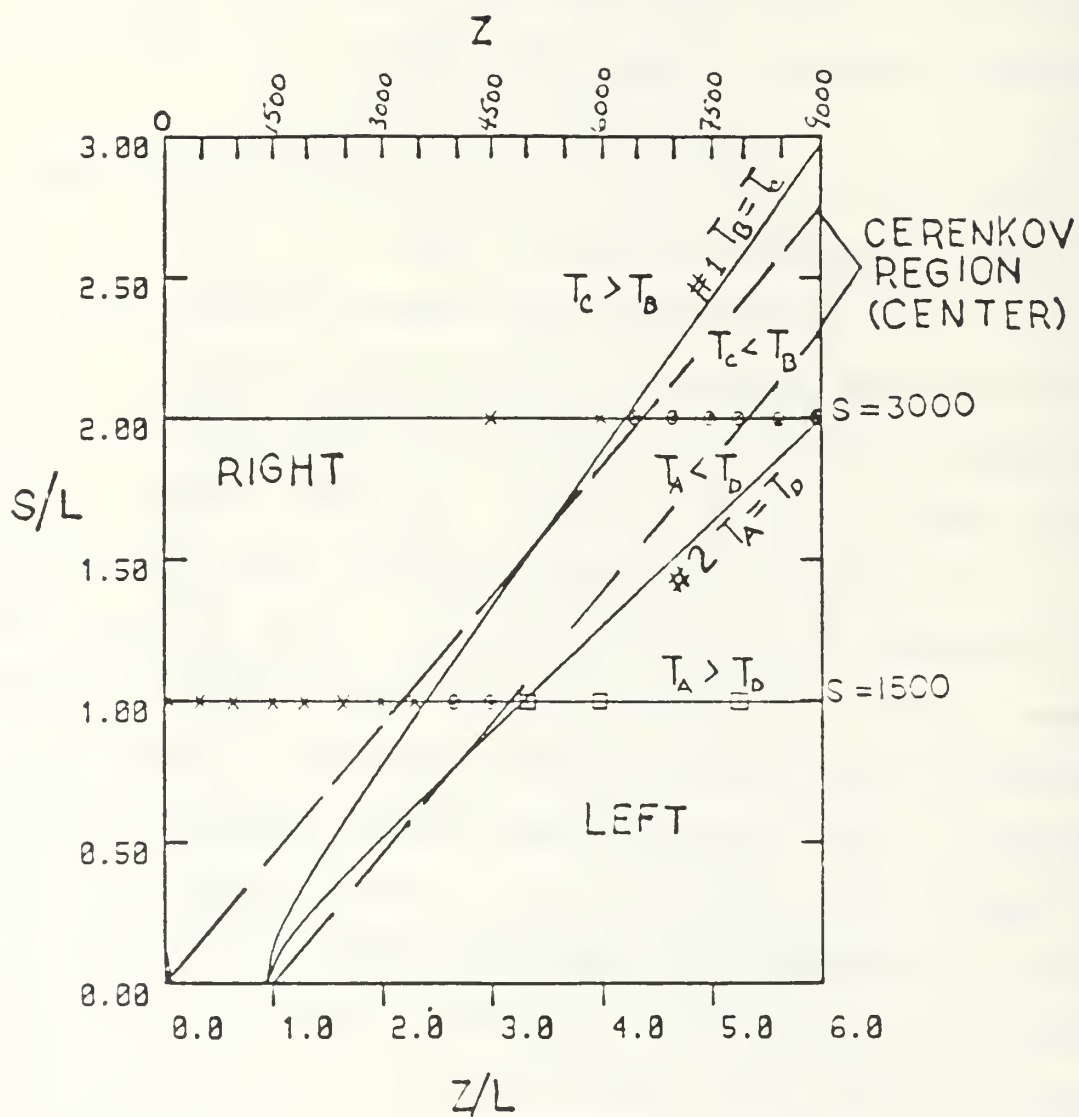


Figure 19: Graph of  $z/L$  vs  $s/L$

increases in magnitude. This is expected, since the radiation should be greatest in the Cerenkov region. The duration of the pulse decreases as  $z$  gets farther from the source ( $L=0$ ); also, it is first seen at a later time.

Figures 29 through 34 (29-31:  $s=1500$ ; 32-34:  $s=3000$ ) show the  $B$  field in the Cerenkov region. The characteristics of the curves shown in Figures 30 through 34 are such that the field increases, levels off and decreases; there are no distinctive peaks. However, Figure 29 shows characteristics of the path to the right, where  $T_c > T_b$ , and  $z$  is close to the Cerenkov region. Although this curve falls into the Cerenkov region, it is also in the time region for  $T_c > T_b$ , to the left of line #1 and is indicated by the symbol  $\times$  in Figure 19. Its shape is dependent on the relationship between  $T_c$  and  $T_b$ . The other graphs in the Cerenkov region fall in the time regions  $T_c < T_b$  and  $T_a < T_b$ , between lines #1 and #2. These are indicated by the symbol  $\odot$  in Figure 19. Other examples of the shape being dependent on the time region are shown in Figures 35 and 36, which indicate a  $B$  field for the path to the right and path to the left, respectively. The shape of these curves is very similar to that of the graphs in the Cerenkov region between lines #1 and #2. The field in Figure 35 falls into the time region  $T_c < T_b$  and the field in Figure 36 falls into the time region  $T_a < T_b$ . Both of these cases are indicated by the symbol  $\odot$  in Figure 19, since they



fall between line #1 and #2. Thus it would seem that the placement of each case in relation to the time boundaries is very important in determining the shape of the B field curve.

Figure 37 (path to the left) shows a field which is extremely close to the time boundary  $T_A = T_D$ . Because this is almost on the boundary, the flat part of the curve is just coming to a point. The other graphs for the path to the left (Figures 38 through 42) show characteristics similar to those of the path to the right. For these graphs, there is an initial peak and then the field falls off; again the magnitude is greater closer to the Cerenkov region. The first time the field is seen increases as  $z$  increases; however, unlike the path to the right, the duration of the field increases slightly as  $z$  increases. This increase in duration would be expected since, in the path to the left, the beam is coming toward the observer and not traveling away. Figures 38 through 42 all fall to the right of line #2, and are indicated by the symbol  $\square$  in Figure 19.

The curves in Figures 20 through 29 all fall in the time region  $T_C > T_B$ . There are four distinct points in these curves: the initial and final points, with two intermediate points. In each of these cases, the points correspond to  $T_A$ ,  $T_B$ ,  $T_C$  and  $T_D$ , in that order. This indicates that the initial rapid increase in the field is

due to the  $u$  function traveling through the distance  $\Delta u$ . This same thing occurs in Figures 38 through 42, in which the time region is  $T_A > T_D$ . However, these points correspond to  $T_C$ ,  $T_D$ ,  $T_A$  and  $T_B$ , the case for the path to the left.

Figures 30 through 33, show B fields for cases which fall well within the Cerenkov region. In each of these cases, there are six distinct points: the initial and final points, with four intermediate points. In Figure 30, for example, these points correspond to  $T_1$ ,  $T_A$ ,  $T_C$ ,  $T_2$ ,  $T_B$  and  $T_3 = T_D$ . Figure 34 shows only four distinct points, corresponding to  $T_C$ ,  $T_A$ ,  $T_D$  and  $T_B$ . This case is almost on the boundary for the Cerenkov region; it shows the four points corresponding to the path to the left, with  $T_A < T_D$ , rather than the six points for the center case.

## V. CONCLUSIONS AND RECOMMENDATIONS

### A. CONCLUSIONS

From this preliminary study, it would seem that the shape of the B field is determined by its placement within a time region, rather than exclusively by the position of the minimum, i.e., path to the right, left or centered about the minimum. In Figure 19, the boundary line #1 and #2 are the cutoffs for determining the characteristic shapes of the B field graphs. Between these two lines, the graphs show a period during which the radiation levels off. On either side of these lines, there is a radiation peak and the field falls off to a secondary peak and then continues to zero. When the study was undertaken, it was expected that the cutoff for characteristic shapes would be the position of the observer in relation to the Cerenkov region. Since the time regions play such an important role, it would be simpler to study the radiation fields from graphs similar to that in Figure 19.

The maximum value of the B field, for a given  $s$  and  $L$ , is found to be in the path centered about the minimum, i.e., in the Cerenkov region.

The duration of the pulse decreases throughout the path to the right and well into the Cerenkov region. This is due to the fact that the beam is traveling away from the observer. For situations occurring close to the boundary

of the Cerenkov region and the path to the left, and continuing into the region for the path to the left, the duration of the pulse increases slightly. For these cases, the beam is traveling toward the observer.

In each case studied, as  $z$  increases, i.e., as the observer gets farther from the source, the pulse first appears at a later time.

It is difficult to predict the characteristics of a curve whose  $z$  value falls on or close to a boundary line (Figure 19), whether that be a time or path boundary. These characteristics include shape, flat topped or peaked, and the number of distinct points on the curve.

In analyzing graphs for different values of  $\Delta u/L$ , it was determined that as  $\Delta u/L$  gets very small, i.e., equals 0.0001, all the time lines coincide. This would mean that all the graphs would have a peaked shape and the flat top curves would disappear, since there would be no region between the time line boundaries.

## B. RECOMMENDATIONS

Since the shape of the B field curve is so closely related to the time regions in Figure 19, it is recommended that several other values of  $\Delta u$  be used for analysis, along with different values for  $s$ , the vertical coordinate of the observer's position,  $P(z,s)$ , in the  $z$ - $s$  plane, and for  $L$ , the beam length.

It is also recommended that some universal time scale be found to use in graphing the radiation fields. For this study, each graph begins at a different time, the time the  $u$  curve begins to transit the rectangle in the  $u$ - $z$  plane. If each graph indicates the same time scale, it would be easier to conduct an analysis and comparison of the radiation fields.

Since there was a problem in correlating the minimum time with the  $B$  field graphs, it is recommended that further study be conducted in this area

In order to make it easier to study the graphs for the case centered about the minimum, it is recommended that the program Fields (Appendix B) be changed so that the print out of the graphs indicate the relationship between  $T_A$  and  $T_D$ , and  $T_B$  and  $T_C$ . Also, a listing of all the appropriate boundary times in each case would be valuable.

## APPENDIX A

FIGURES: B FIELD CURVES

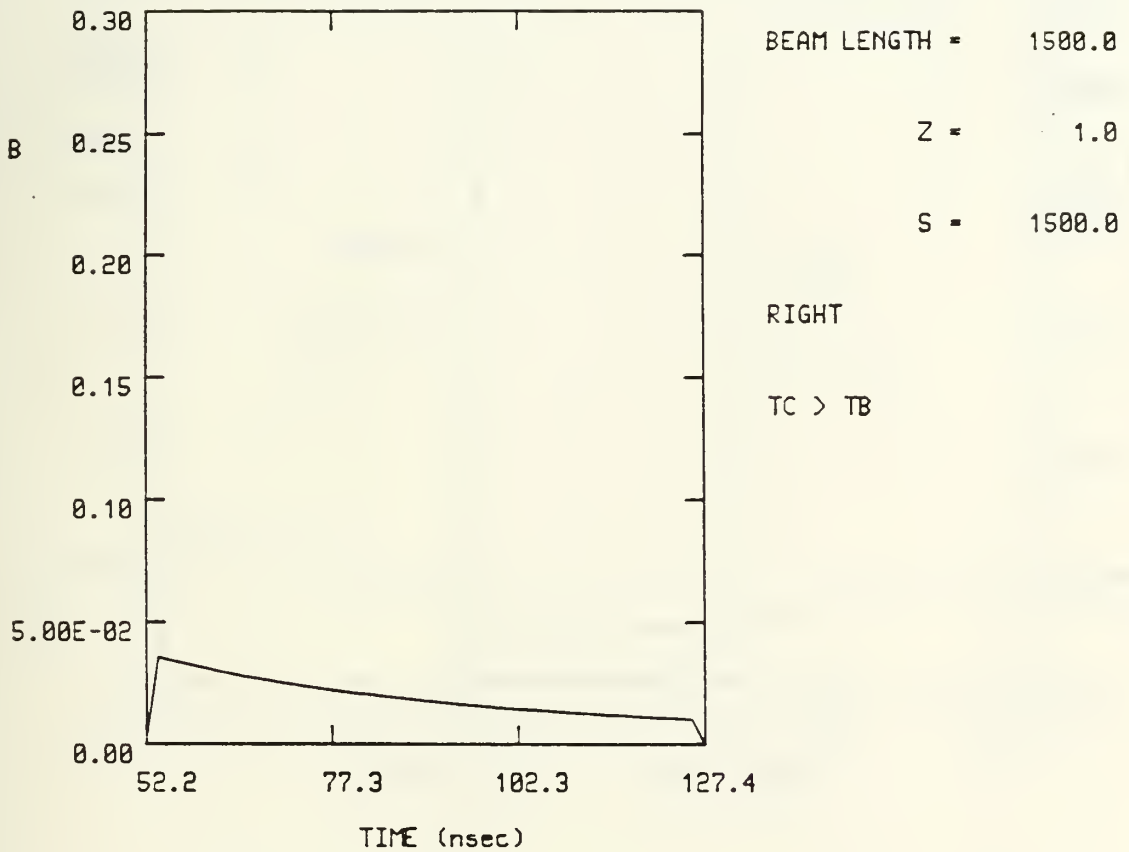


Figure 20:  $s=1500$ ,  $z=1$ ;  $T_c > T_b$

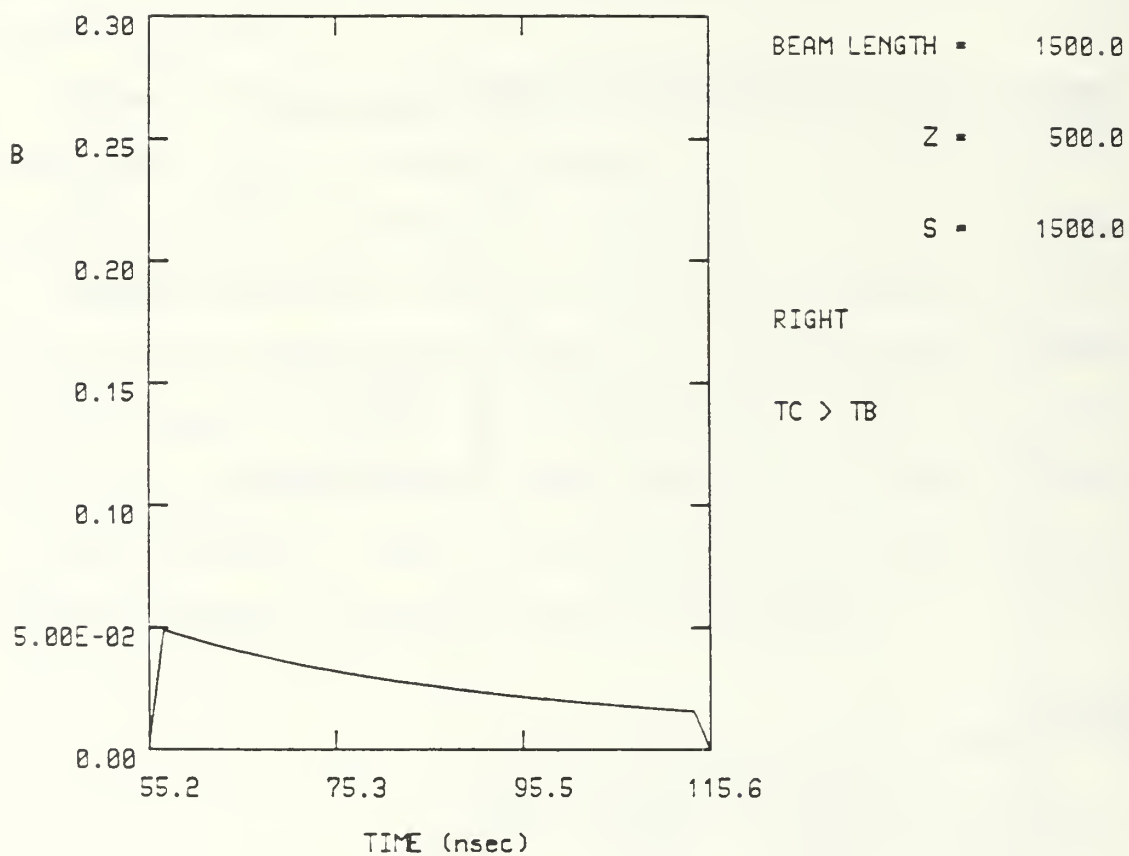


Figure 21:  $s=1500$ ,  $z=500$ ;  $T_c > T_b$



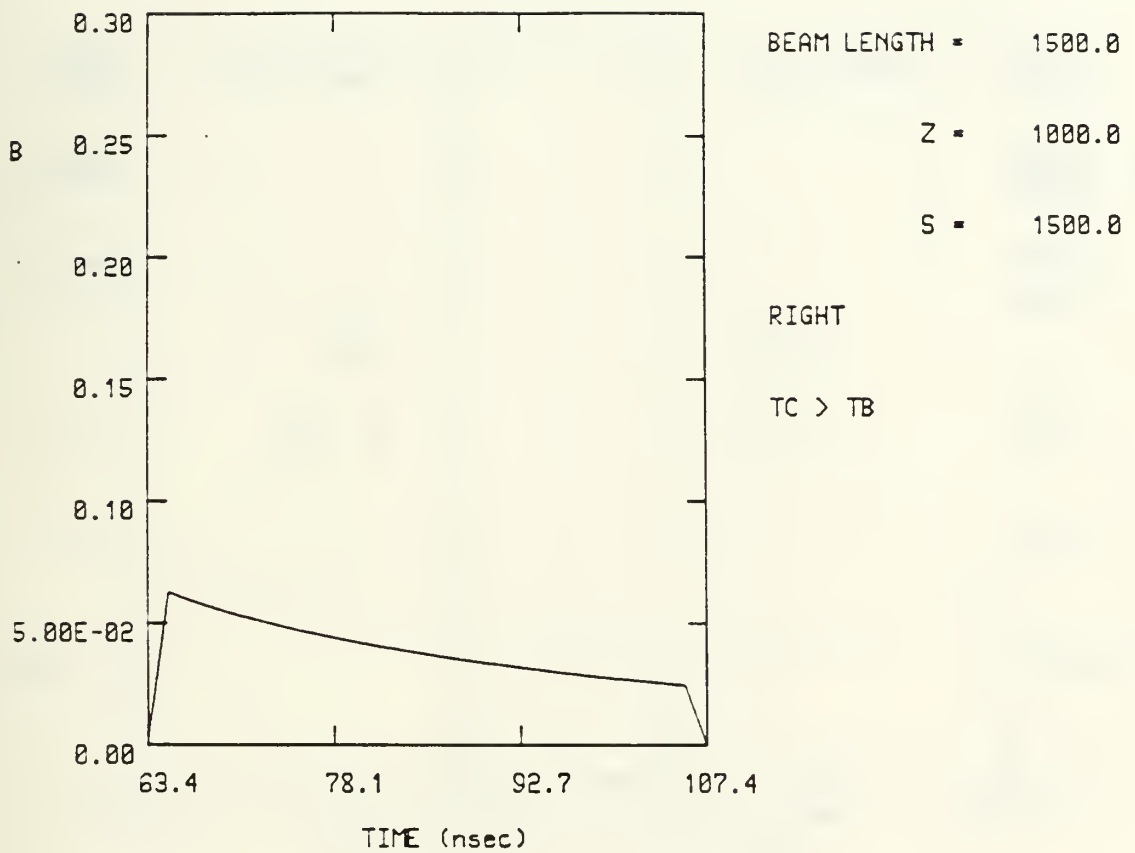


Figure 22:  $s=1500$ ,  $z=1000$ ;  $T_c > T_b$

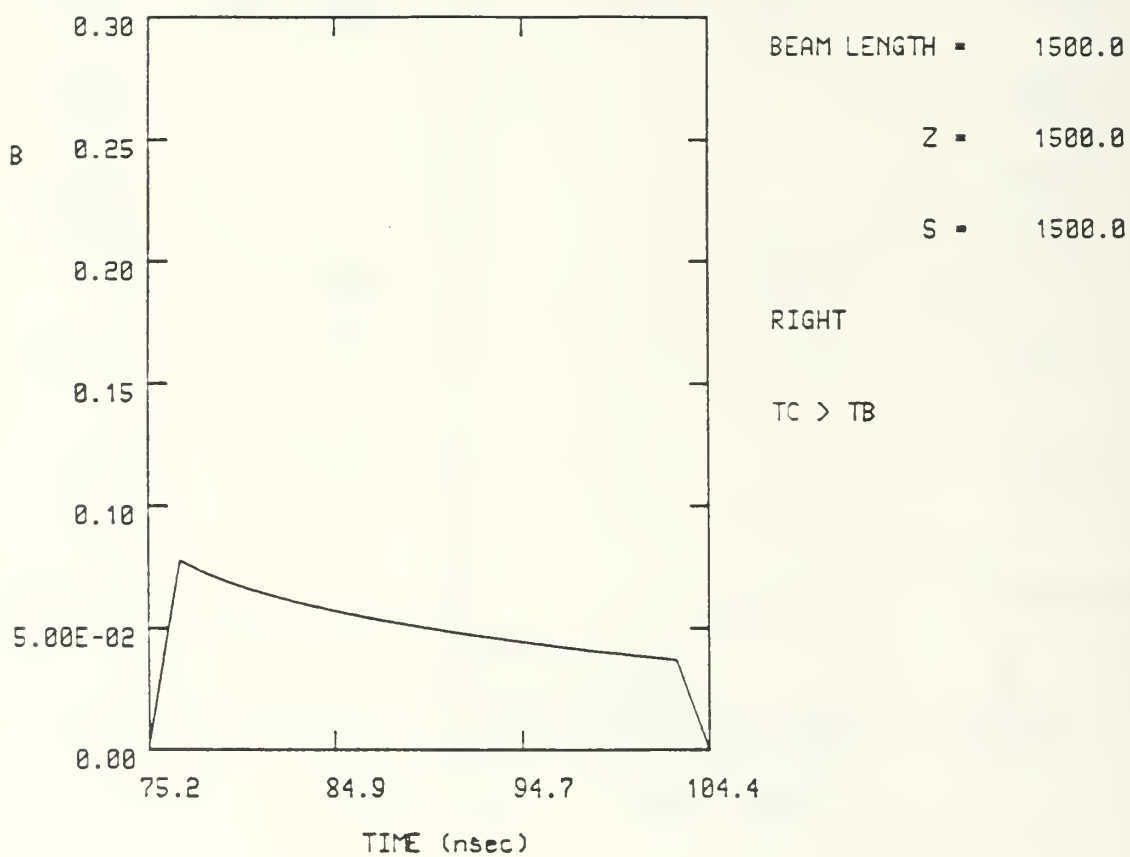


Figure 23:  $s=1500$ ,  $z=1500$ ;  $T_c > T_b$

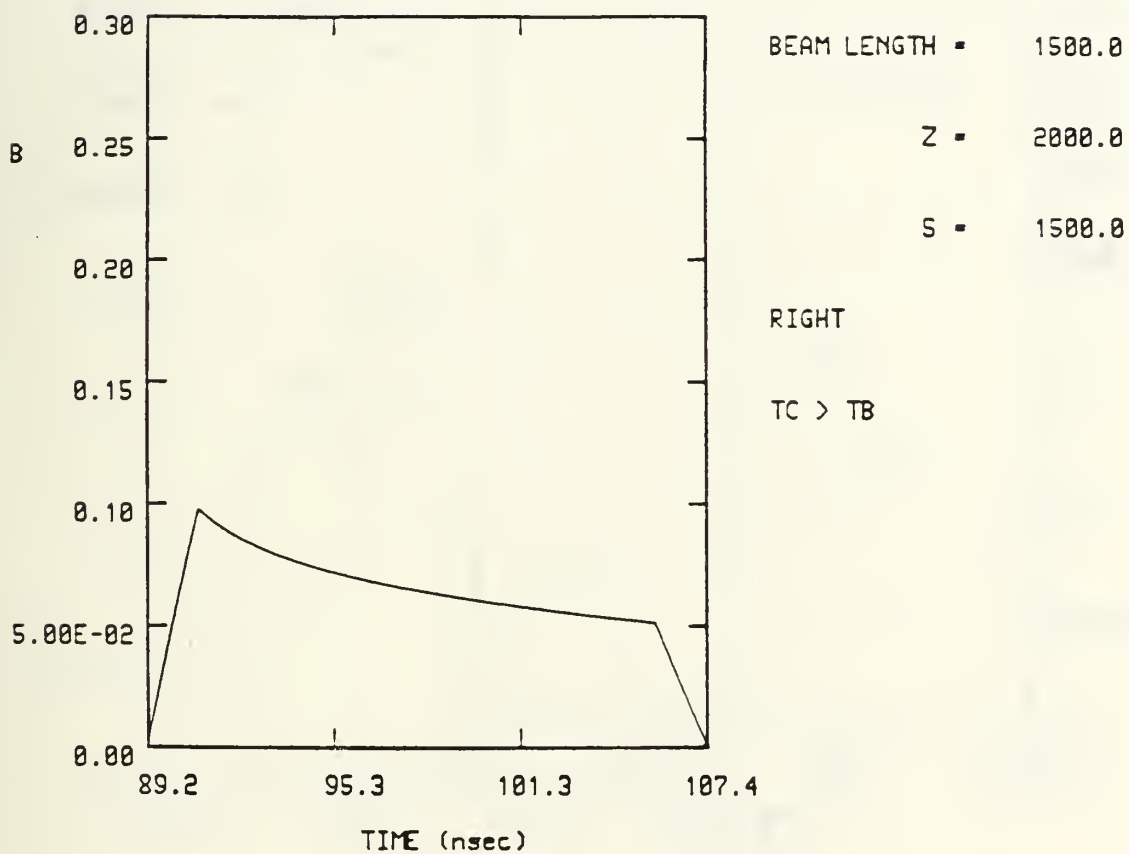


Figure 24:  $s=1500$ ,  $z=2000$ ;  $T_c > T_b$

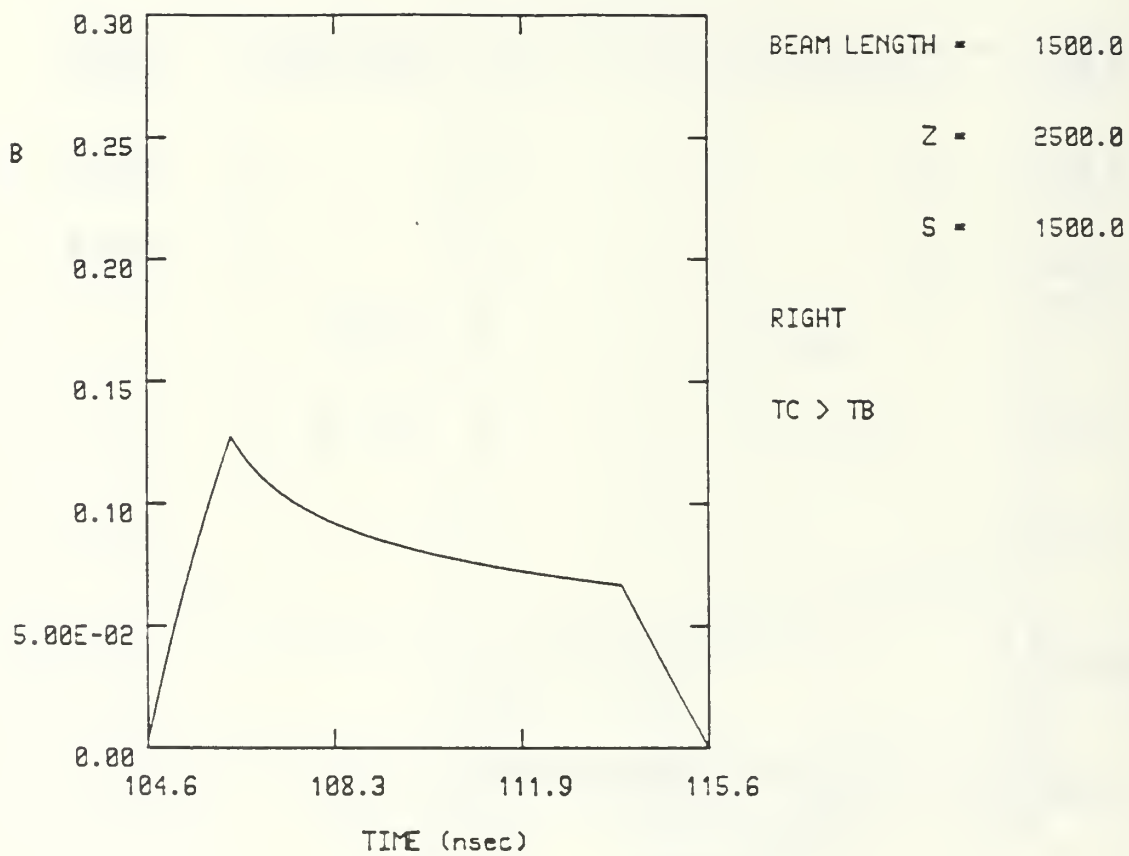


Figure 25:  $s=1500$ ,  $z=2500$ ;  $T_c > T_b$

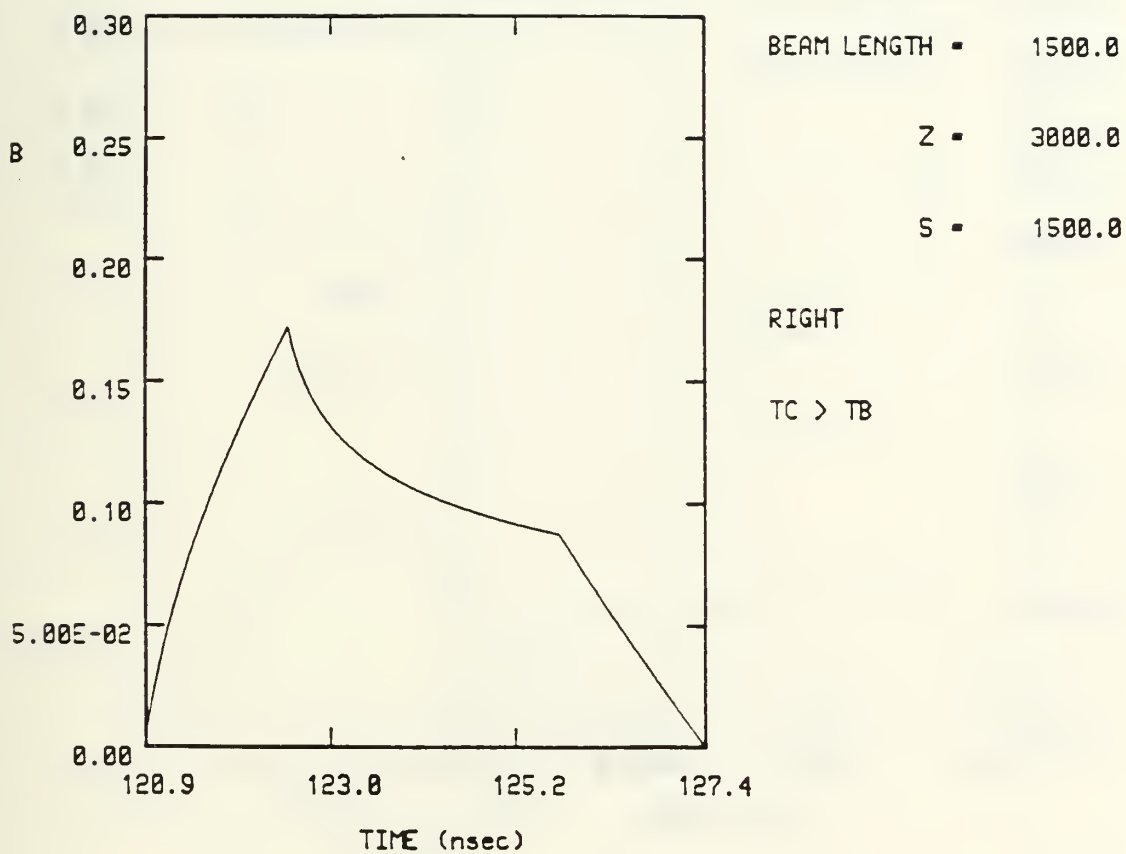


Figure 26:  $s=1500$ ,  $z=3000$ ;  $T_c > T_b$

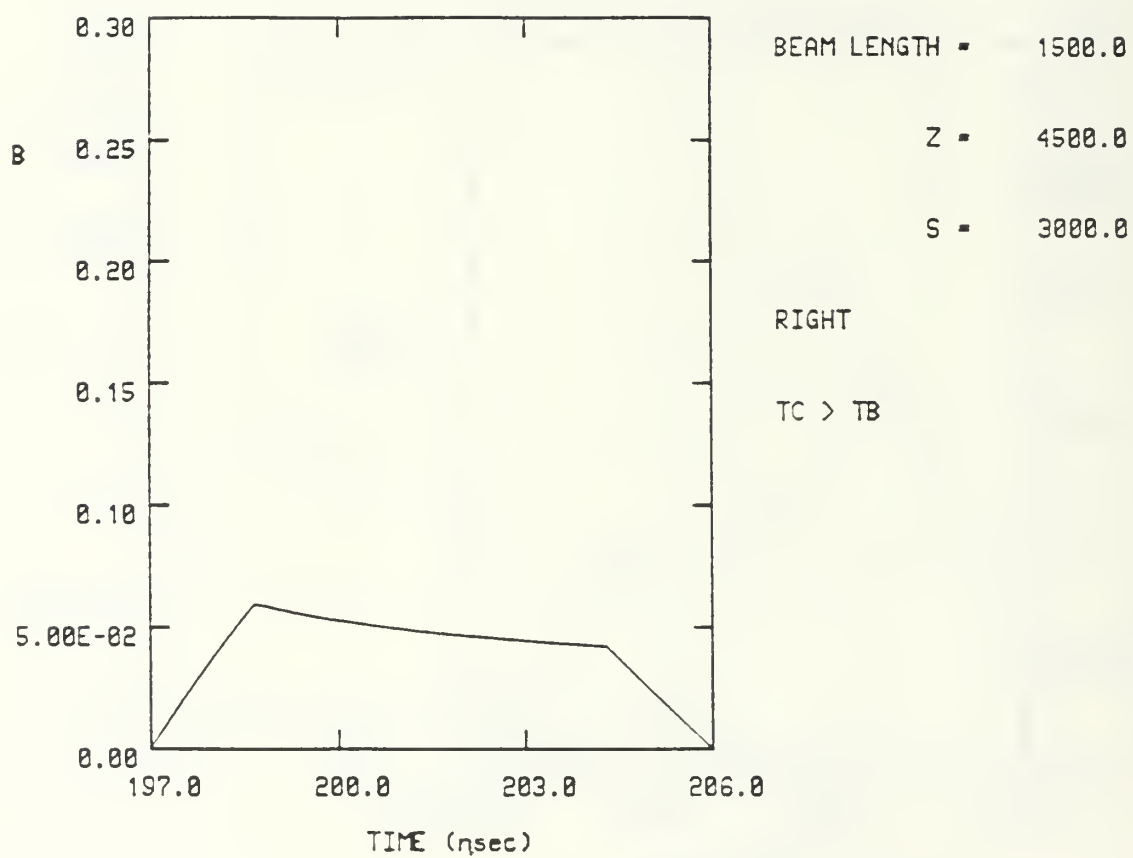


Figure 27:  $s=3000$ ,  $z=4500$ ;  $T_c > T_b$

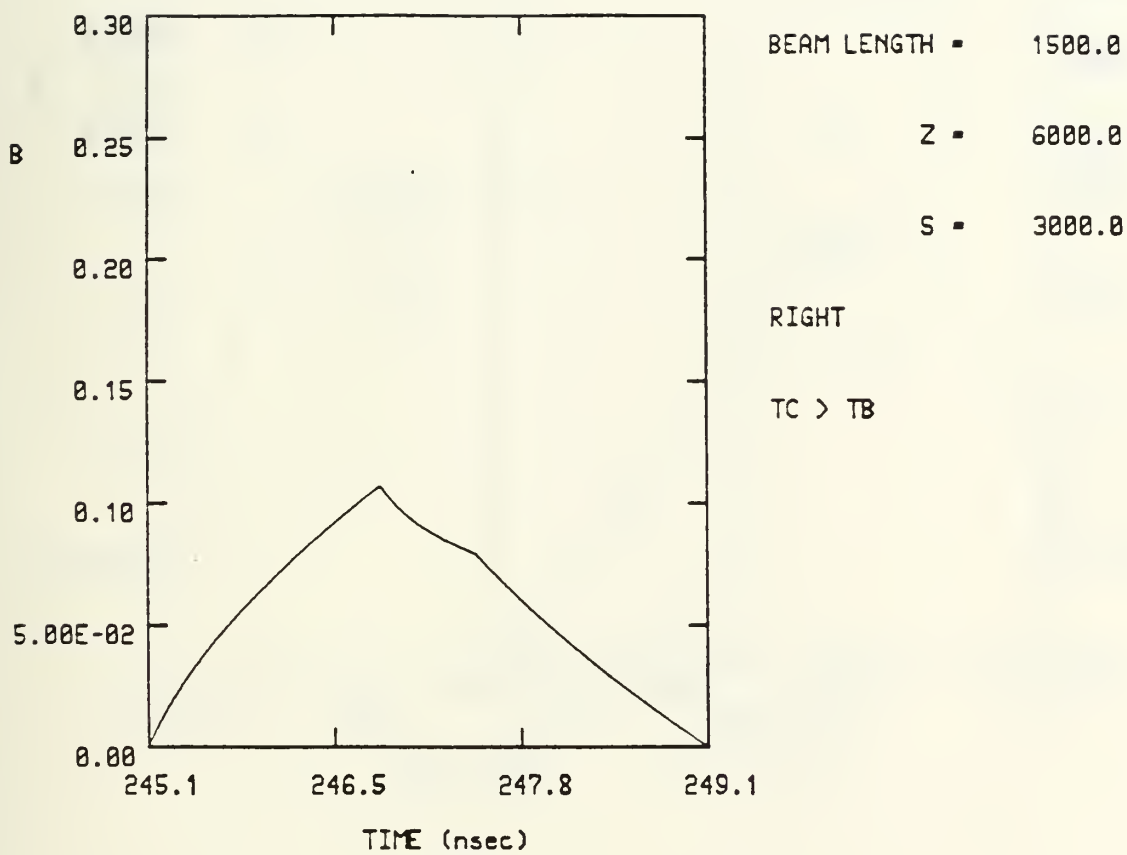


Figure 28:  $s=3000$ ,  $z=6000$ ;  $T_c > T_b$



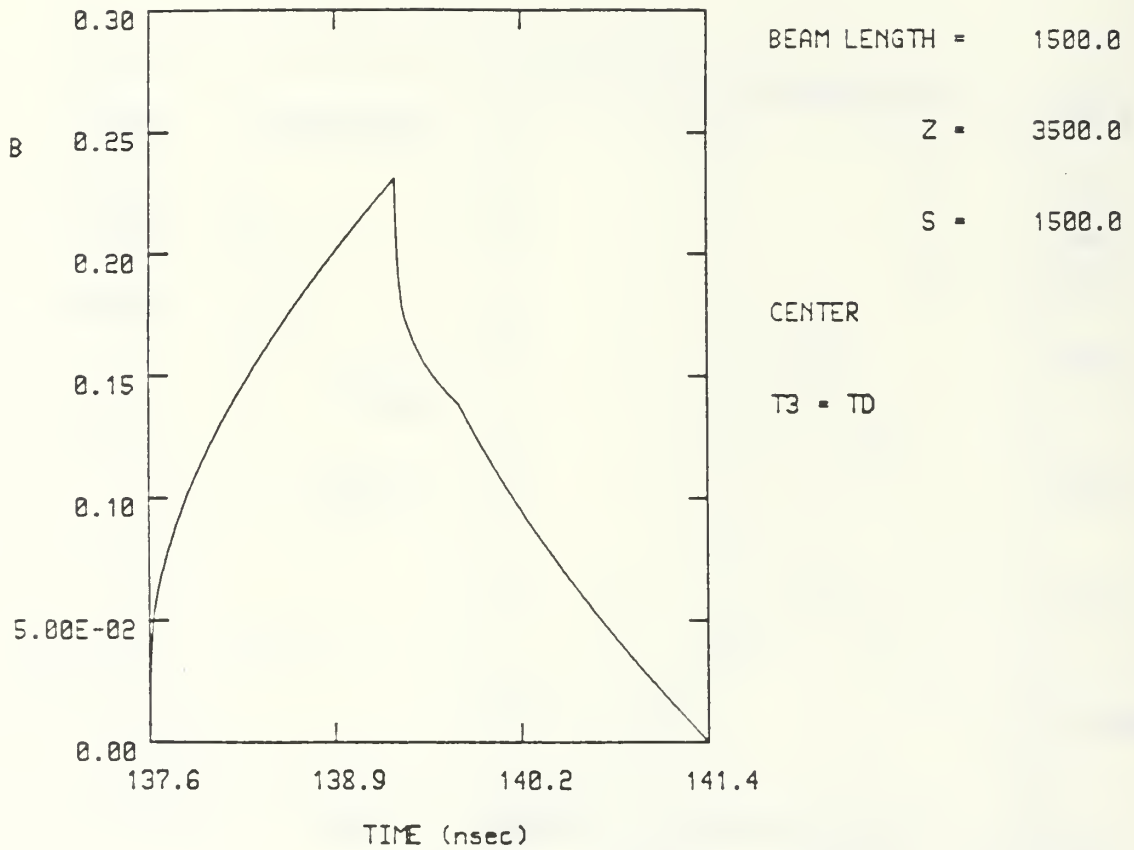


Figure 29:  $s=1500$ ,  $z=3500$ ;  $T_c > T_b$

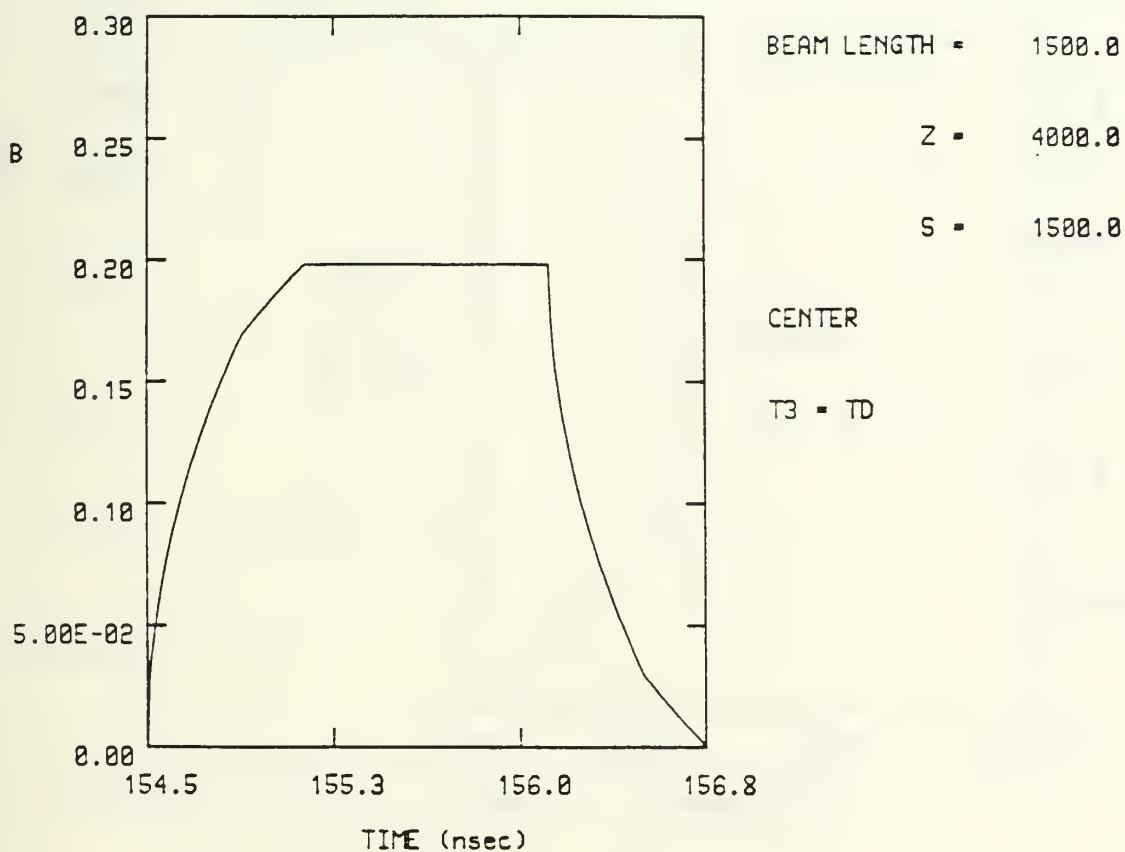


Figure 30:  $s=1500$ ,  $z=4000$ ; Time Region Between Lines  
#1 and #2 of Figure 19

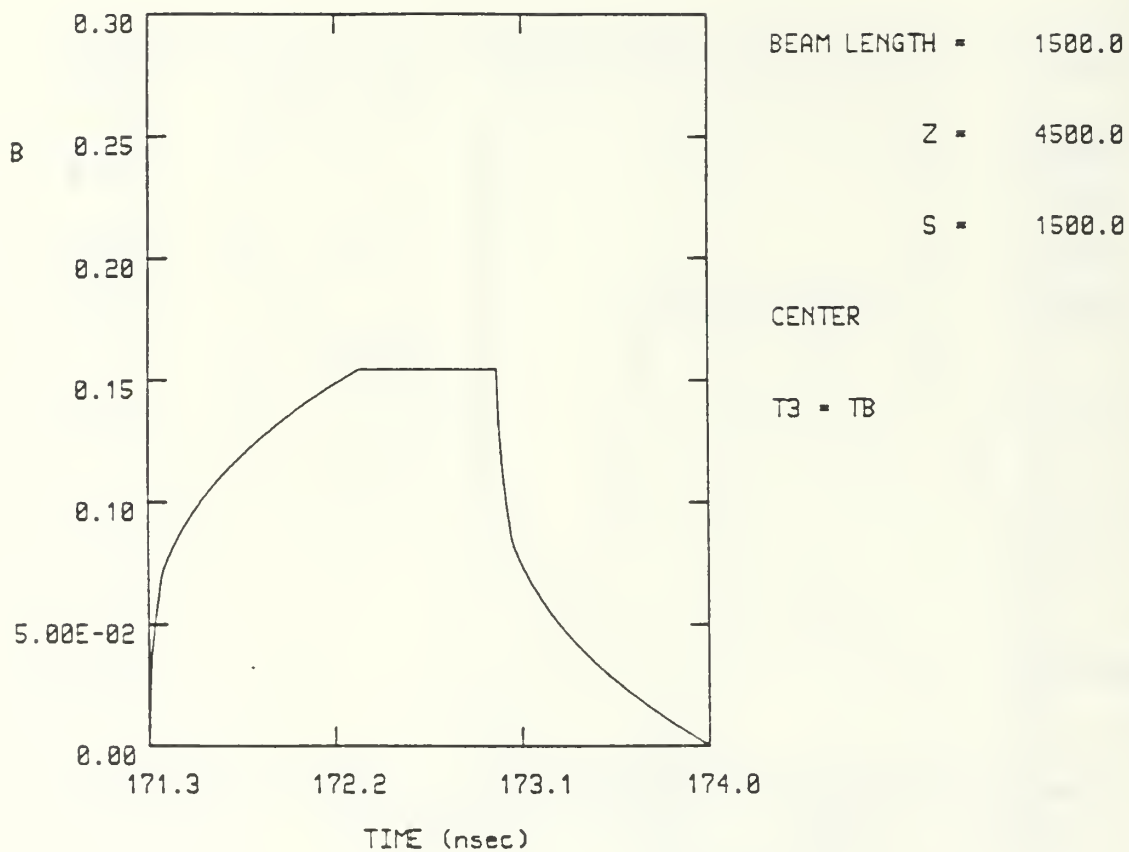


Figure 31:  $s=1500$ ,  $z=4500$ ; Time Region Between Lines  
#1 and #2 of Figure 19

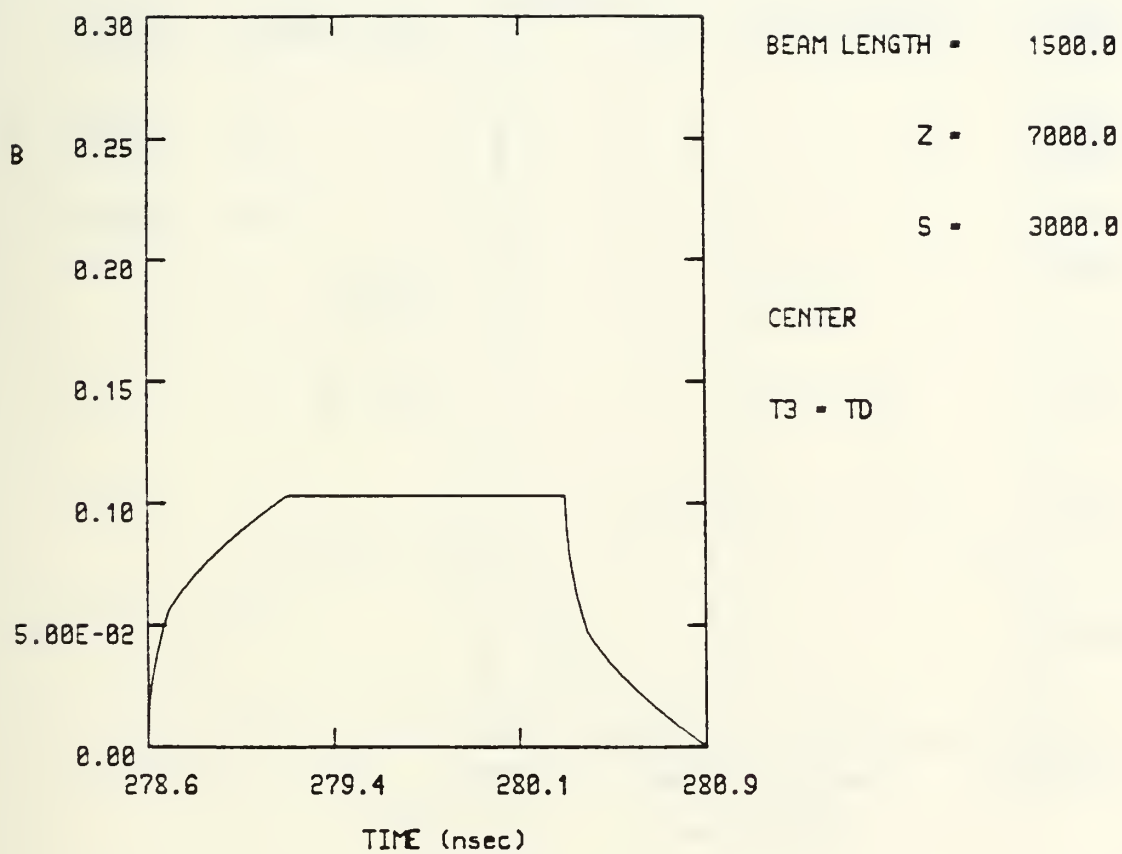


Figure 32:  $s=3000$ ,  $z=7000$ ; Time Region Between Lines  
#1 and #2 of Figure 19

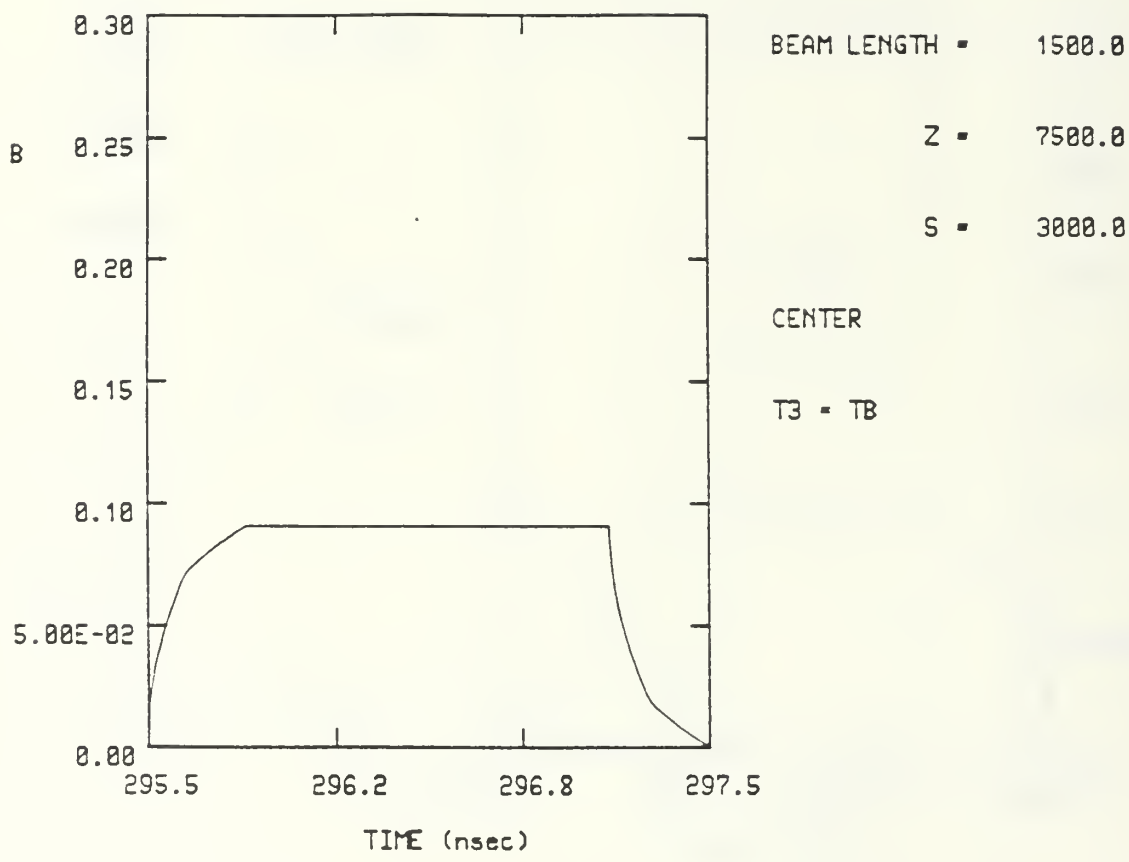


Figure 33:  $s=3000$ ,  $z=7500$ ; Time Region Between Lines #1 and #2 of Figure 19

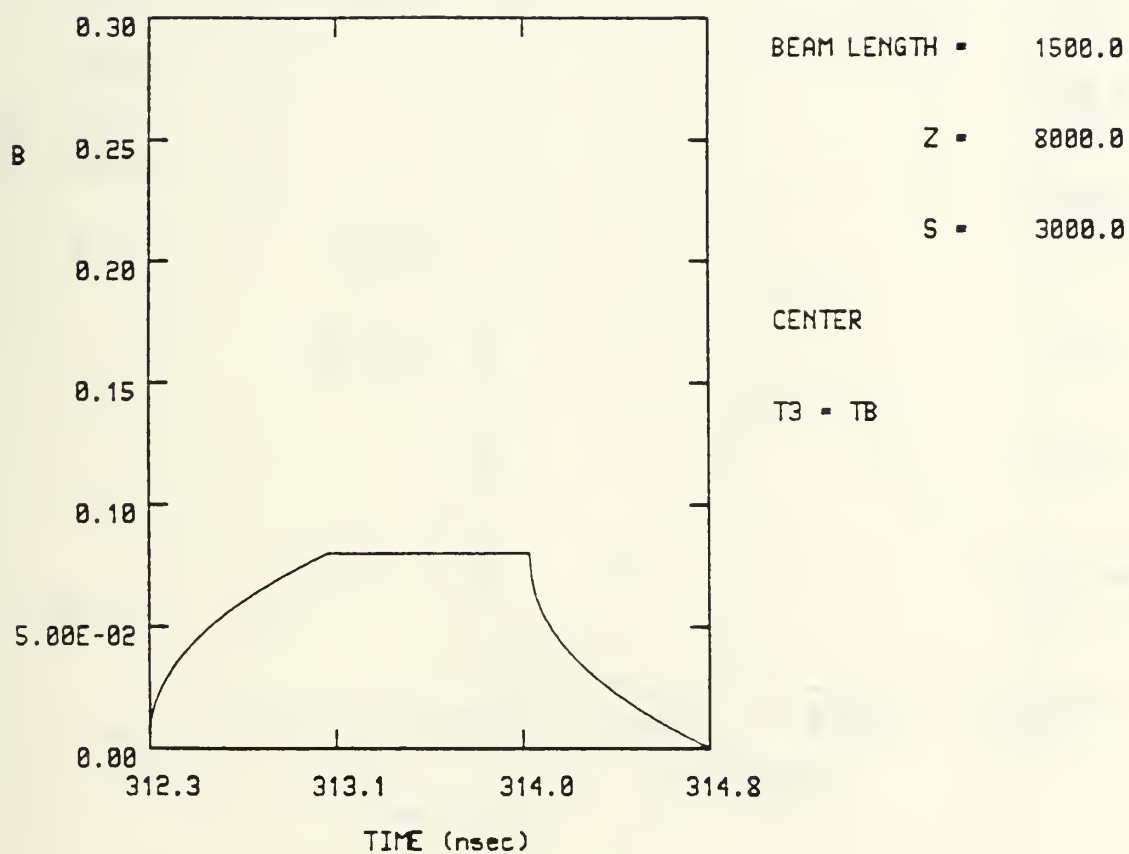


Figure 34:  $s=3000$ ,  $z=8000$ ; Time Region Between Lines #1 and #2 of Figure 19

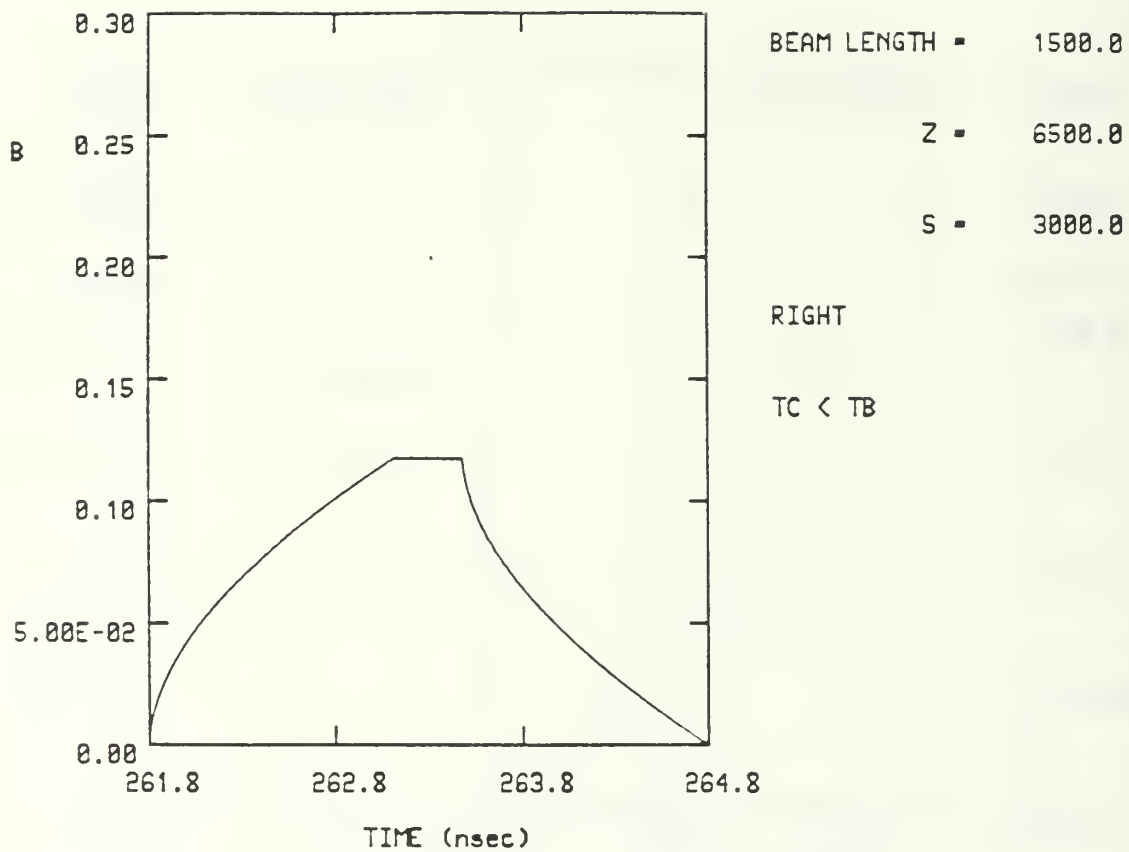


Figure 35:  $s=3000$ ,  $z=6500$ ; Time Region Between Lines  
#1 and #2 of Figure 19



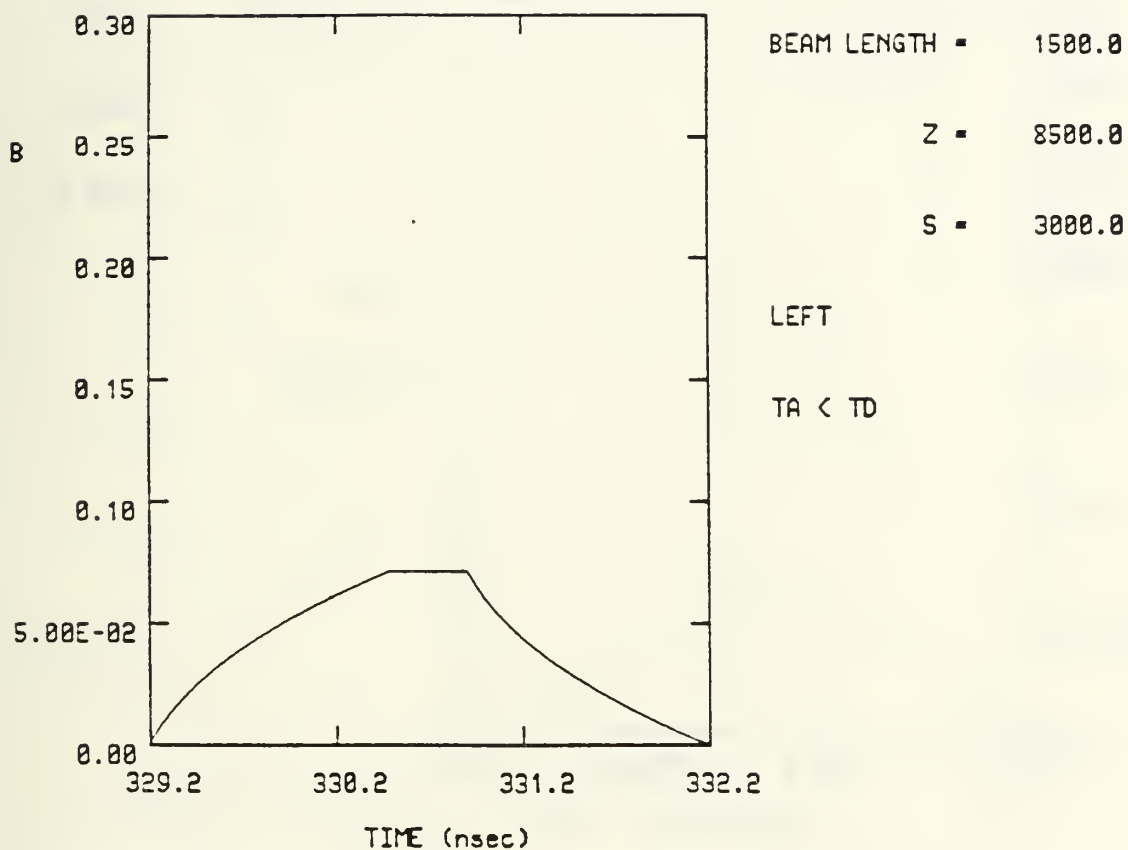


Figure 36:  $s=3000$ ,  $z=8500$ ; Time Region Between Lines #1 and #2 of Figure 19

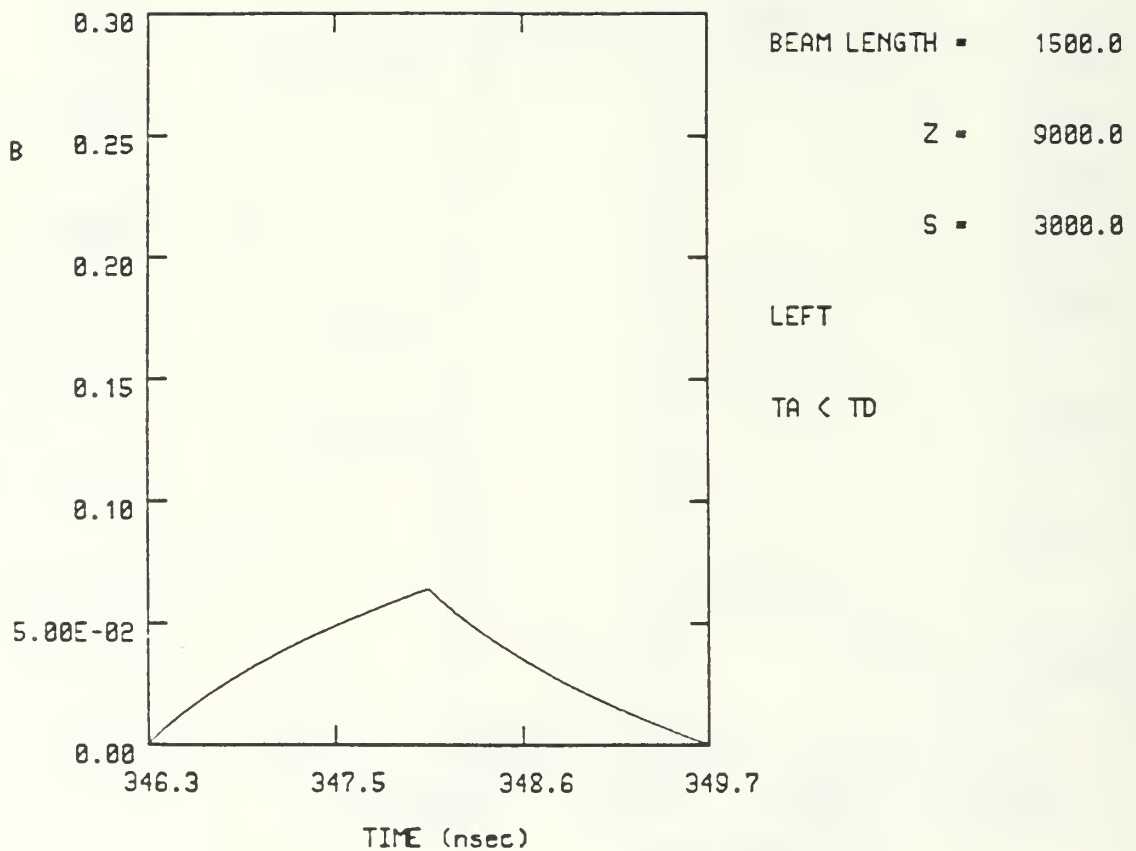


Figure 37:  $s=3000$ ,  $z=9000$ ; Time Region Between Lines  
#1 and #2 of Figure 19

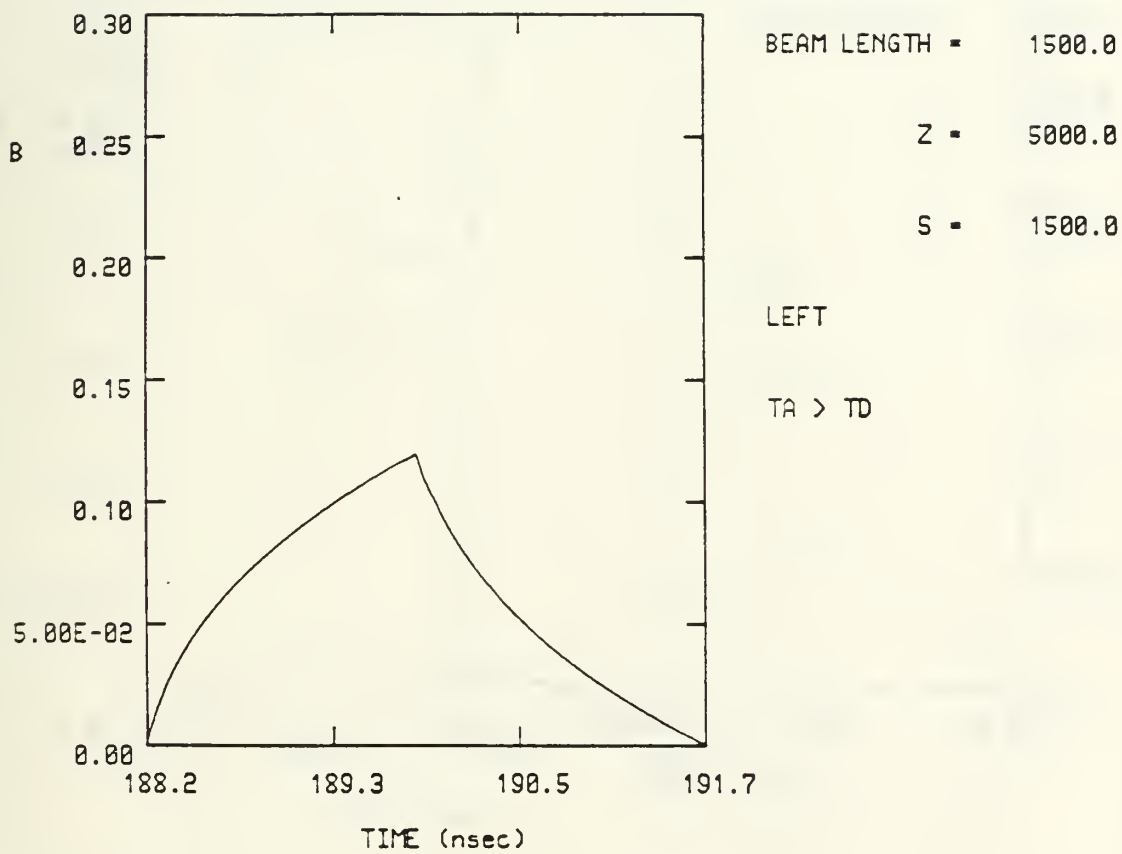


Figure 38: s=1500, z=5000; TA>Td

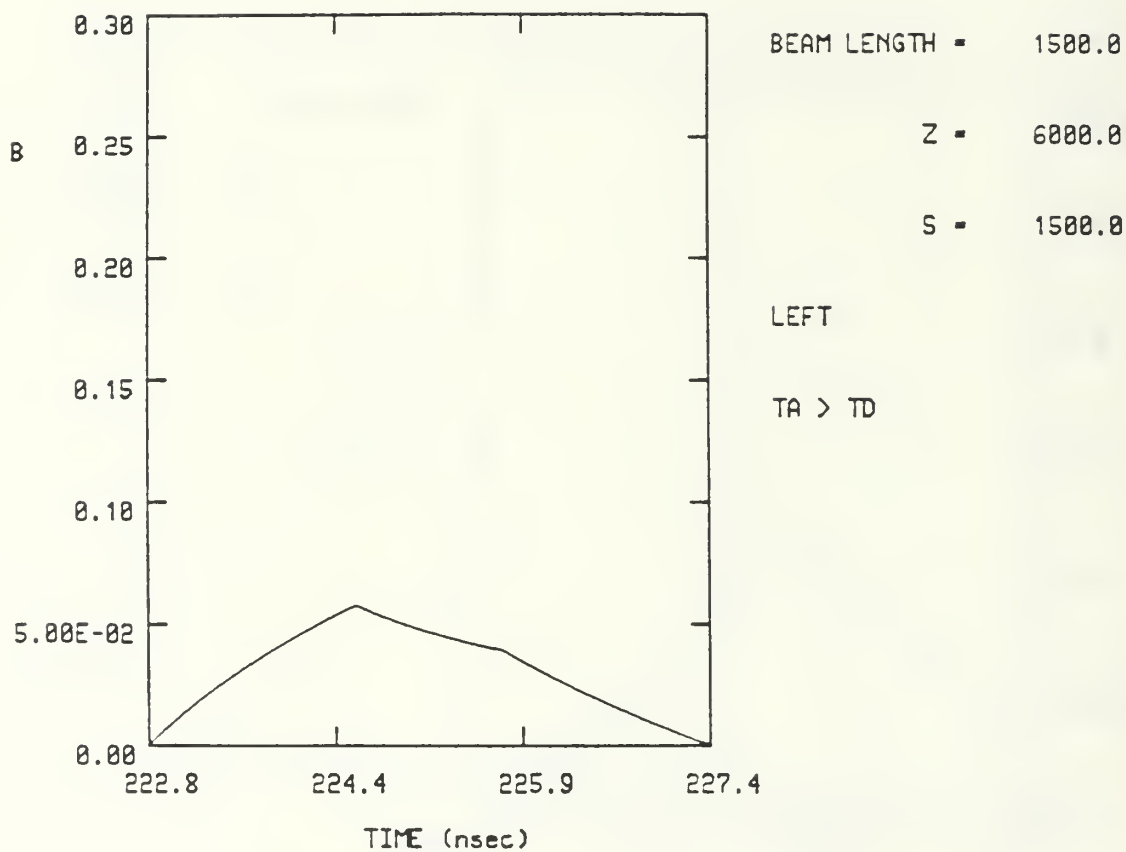


Figure 39: s=1500, z=6000; TA>Td

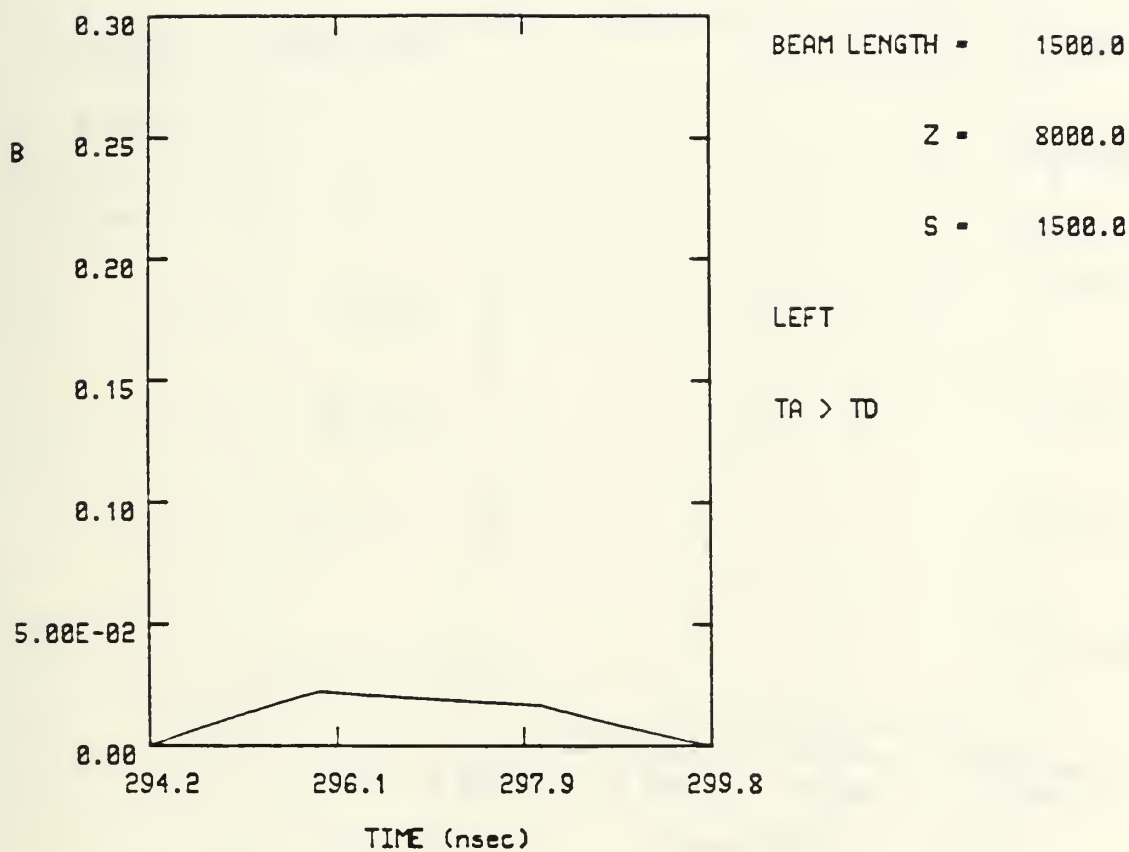


Figure 40: s=1500, z=8000; TA>TD

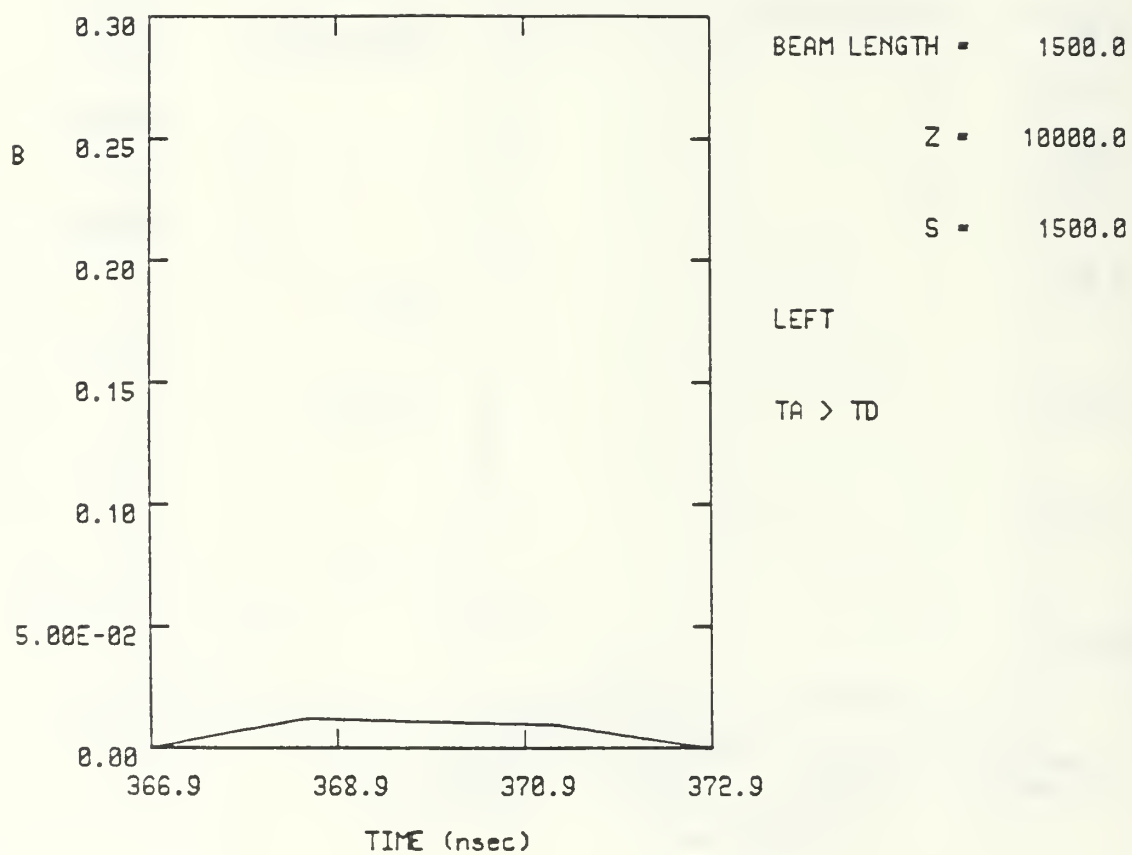


Figure 41:  $s=1500$ ,  $z=10000$ ;  $T_A > T_D$

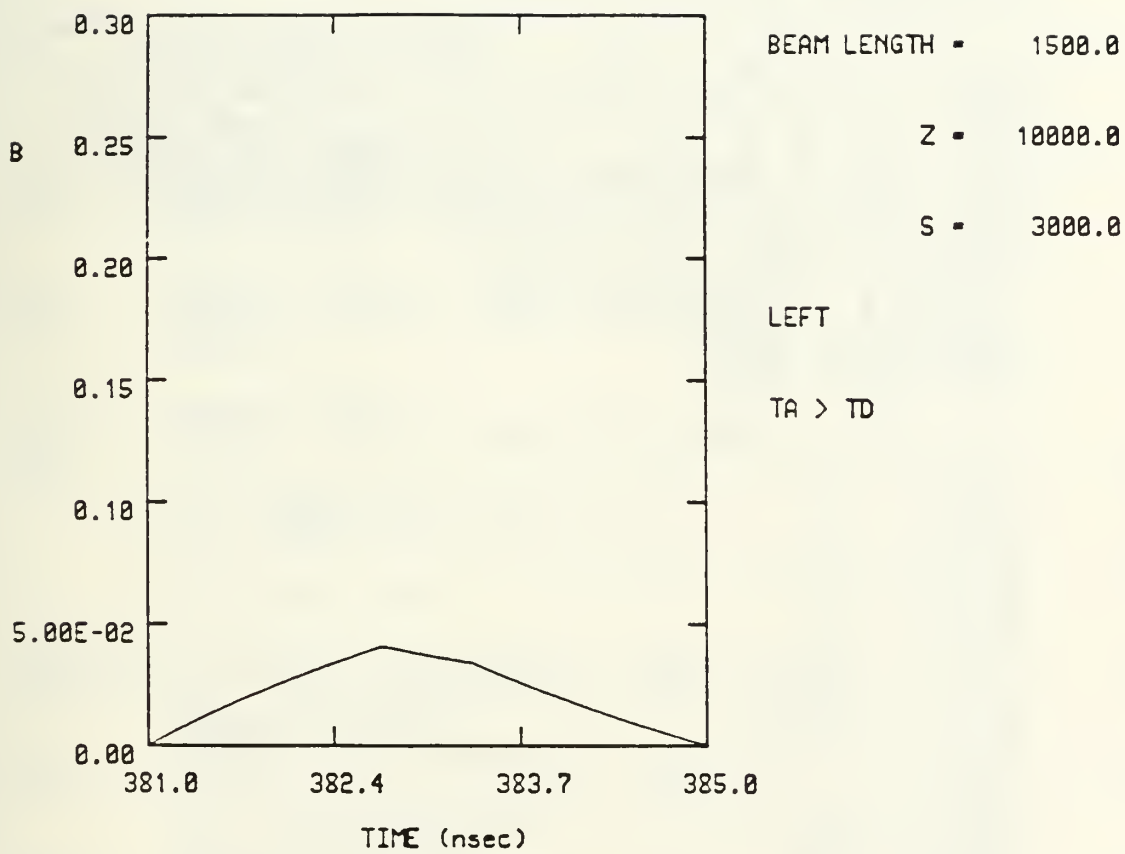


Figure 42:  $s=3000$ ,  $z=10000$ ;  $TA > TD$



## APPENDIX B

### FORTTRAN PROGRAM: FIELDS

C\*\*\*\*\* PROGRAM FIELDS \*\*\*\*\*

C\*\*\*\*\* THIS PROGRAM WILL CALCULATE THE FIELDS FROM A  
PASSING CHARGE BUNCH. IT WAS WRITTEN BY LCDR  
KATHLEEN M. LYMAN, USN; THE GRAPHICS PORTION OF THE  
PROGRAM IS BASED ON CODE WRITTEN BY PROF. J.R.  
NEIGHBOURS OF THE NAVAL POSTGRADUATE SCHOOL.

```
REAL N,U1,U2,BETA,CO,ROE,A,G,CE,V,BPRME,R1,R2,A1
REAL A2,D,E,DD,EE,Q,TA,TB,TC,TD,F,ZPI,ZPF,WPI,WPF
REAL W1,W2,ZC,RC,ZPC,T1,T2,T3,DELR,ZPM,ZPM2,B1,B2
REAL E1,E2,S1,XX,YY,S,L,Z,TPRME,B,BMAX,TMAX,XMAX
REAL SDYY,SDXX,SCALEY,SCALEX,SDX,SDY,SN,YN,YMIN,X,Y
REAL YMAX,XMIN,TMIN,GPRME,THETA1,THETA2
DIMENSION TPRME(9000),B(9000)
INTEGER I,J,JXMAX,JYMAX,IMAX
CHARACTER*1 AXCH,PRE
CHARACTER*6 PATH
CHARACTER*8 TIME
```

```
300      WRITE(6,2000)
2000     FORMAT('  U1 = ', $)
        READ(5,*)U1
        WRITE(6,2001)
2001     FORMAT('  U2 = ', $)
        READ(5,*)U2
        WRITE(6,2005)
2005     FORMAT(' BETA = ', $)
        READ(5,*)BETA
        WRITE(6,2007)
2007     FORMAT('   N = ', $)
        READ(5,*)N
        WRITE(6,2009)
2009     FORMAT('  ROE = ', $)
        READ(5,*)ROE
```

C\*\*\*\*\* INITIAL VALUES \*\*\*\*\*

```
DO 900 I = 1,9000
TPRME(I) = 0.0
900      CONTINUE
```

```
DO 910 I = 1,9000
B(I) = 0.0
910      CONTINUE
```

```

      TMAX=0.0
      TMIN=0/0
      IMAX=0
      BMAX=0.00000000
      C0=29.997250

C***** CALCULATE THE VELOCITY V AND BETA PRIME (BPRME)
      V=BETA*C0
      BPRME=N*BETA

      Q=BPRME**2.-1.

C***** CALCULATE THE CERENKOV ANGLE (CE) *****
      CE = ACOS(1/BPRME)
      F = TAN(CE)

      WRITE(6,1002)
1002   FORMAT('ENTER S VALUE FOR GRAPH      ', $)
      READ(5,*)S

      WRITE(6,1003)
1003   FORMAT('ENTER L VALUE FOR GRAPH      ', $)
      READ(5,*)L

      WRITE(6,1004)
1004   FORMAT('ENTER Z VALUE FOR GRAPH      ', $)

      R1 = SQRT(S**2.+Z**2.)
      R2 = SQRT(S**2.+(Z-L)**2.)

      S1 = (S**2.)*Q

C***** CALCULATE THE BOUNDARY TIMES *****
      TA = (BPRME*R1-U1)/V
      WRITE(6,3013)TA
3013   FORMAT('TA: ', 4X, F9.4)

      TB = (BPRME*R1-U2)/V
      WRITE(6,3014)TB
3014   FORMAT('TB: ', 4X, F9.4)

      TC = (L+(BPRME*R2)-U1)/V
      WRITE(6,3015)TC
3015   FORMAT('TC: ', 4X, F9.4)

      TD = (L+(BPRME*R2)-U2)/V
      WRITE(6,3016)TD
3016   FORMAT('TD: ', 4X, F9.4)

```

```

C***** CALCULATE THE VALUE OF TAN(THETA 1),(A), AND
C***** THE VALUE OF TAN(THETA 2),(G)
      A=S/Z
      IF(Z.LT.L)GO TO 500
      IF(Z.EQ.L)GO TO 501
      G=S/(Z-L)
      GO TO 502

C***** COMPARISON TO DETERMINE ON WHICH SIDE THE MINIMUM
C***** LIES
500      GPRME=ATAN((L-Z)/S)+90.
      THETA1=ATAN(A)
      THETA2=ATAN(GPRME)
      IF(THETA1.GT.CE)GO TO 10
      IF(THETA2.GE.CE)GO TO 15
      GO TO 20

501      GPRME=90.
      THETA1=ATAN(A)
      IF(THETA1.GT.CE)GO TO 10
      GO TO 15

502      IF(A.GT.F)GO TO 10
      IF(G.GE.F)GO TO 15
      GO TO 20

C***** PATH TO THE RIGHT *****

10      WRITE(6,2050)
2050     FORMAT('PATH TO THE RIGHT')
      PATH='RIGHT'
      TMIN=TA

303      DO 701 I=1,9000
      TPRME(I)=TA+(REAL(I))/100.
      IF(TPREM(I).GE.TD)GO TO 800

      TMAX=MAX(TMAX,TPRME(I))
      IMAX=MAX(IMAX,I)

      A1=U1+V*TPRME(I)
      A2=U2+(V*TPRME(I))
      D=((BPRME**2.)*Z)-A1
      DD=((BPRME**2.)*Z)-A2
      E1=((Z-A1)**2.)-S1
      E2=((Z-A2)**2.)-S1

      IF(TC.GT.TB)GO TO 25
      IF(TC.LT.TB)GO TO 30
      IF(TC.EQ.TB)GO TO 35

```

```

C***** TC > TB:  CALCULATION OF LIMITS OF INTEGRATION
25  TIME='TC >TB'
    IF((TA.LT.TPRME(I)).AND.(TPRME(I).LT.TB))GO TO 26
    IF((TB.LT.TPRME(I)).AND.(TPRME(I).LT.TC))GO TO 27
    IF((TC.LT.TPRME(I)).AND.(TPRME(I).LT.TD))GO TO 28

C***** TA <T'<TB
26  ZPI=0.
    IF(E1.LT.0.)GO TO 200
    E=BPRME*SQRT(E1)
    ZPF=(D+E)/Q
    IF(ZPF.LT.0.)GO TO 303
    GO TO 101

C***** TB<T'<TC
27  IF(E1.LT.0.)GO TO 200
    E=BPRME*SQRT(E1)
    IF(E2.LT.0.)GO TO 220
    EE=BPRME*SQRT(E2)
    ZPI=(DD+EE)/Q
    ZPF=(D+E)/Q
    IF(ZPI.LT.0.)GO TO 303
    IF(ZPF.LE.0.)GO TO 303
    GO TO 101

C***** TC<T'<TD
28  IF(E2.LT.0.)GO TO 220
    EE=BPRME*SQRT(E2)
    ZPI=(DD+EE)/Q
    ZPF=L
    IF(ZPI.LT.0.)GO TO 303
    GO TO 101

C***** TC < TB:  CALCULATION OF LIMITS OF INTEGRATION
30  TIME='TC < TB'
    IF((TA.LT.TPRME(I)).AND.(TPRME(I).LT.TC))GO TO 31
    IF((TC.LT.TPRME(I)).AND.(TPRME(I).LT.TB))GO TO 32
    IF((TC.LT.TPRME(I)).AND.(TPRME(I).LT.TD))GO TO 33

C***** TA<T'<TC
31  ZPI=0
    IF(E1.LT.0.)GO TO 200
    E=BPRME*SQRT(E1)
    ZPF=(D+E)/Q
    IF(ZPF.LT.0.)GO TO 303
    GO TO 101

C***** TC<T'<TB
32  ZPI=0.
    ZPF=L
    GO TO 101

```

```

C***** TB<T'<TD
33      IF(E2.LT.0.)GO TO 220
        EE=BPRME*SQRT(E2)
        ZPI=(DD+EE)/Q
        ZPF=L
        IF(ZPI.LT.0.)GO TO 303
        GO TO 101

C***** TC = TB:  CALCULATION OF LIMITS OF INTEGRATION
35      TIME='TC = TB'
        IF((TA.LT.TPRME(I)).AND.(TPRME(I).LT.TB))GO TO 36
        IF((TB.LT.TPRME(I)).AND.(TPRME(I).LT.TD))GO TO 37

C***** TA<T'<TB
36      ZPI=0.
        IF(E1.LT.0.)GO TO 200
        E=BPRME*SQRT(E1)
        ZPF=(D+E)/Q
        IF(ZPF.LT.0.)GO TO 303
        GO TO 101

C***** TB<T'<TD
37      IF(E2.LT.0.)GO TO 220
        EE=BPRME*SQRT(E2)
        ZPI=(DD+EE)/Q
        ZPF=L
        IF(ZPF.LT.0.)GO TO 303
        GO TO 101

C***** CALCULATION OF THE FIELD
101     WPI=Z-ZPI
        WPF=Z-ZPF
        W1=WPI/S
        W2=WPF/S
        YY=ATAN(W1)
        XX=ATAN(W2)
        B(I)=(ROE*N*(BETA**2))*(YY-XX)
        BMAX=MAX(BMAX,B(I))
701     CONTINUE
        GO TO 800

C***** PATH CENTERED ABOUT THE MINIMUM *****
15      WRITE(6,2051)
2051     FORMAT('CENTER')
        PATH='CENTER'
        ZPC=Z-(S/F)
        RC=SQRT(S**2.+((Z-ZPC)**2.))

C***** CALCULATION OF T1 AND T2
        T1=(ZPC+(BPRME*RC)-U1)/V
        T2=(ZPC+(BPRME*RC)-U2)/V
        DELR=R1-R2

```

```

        WRITE(6,6000)T1
6000    FORMAT('T1= ',F9.4)
        WRITE(6,6001)T2
6001    FORMAT('T2= ',F9.4)

C***** DETERMINING THE VALUE OF T3
        IF((BPRME*DELR).GT.L)GO TO 16
        IF((BPRME*DELR).LE.L)GO TO 17
16      T3=TB
        TIME='T3 = TB'
        GO TO 18
17      T3=TD
        TIME='T3 = TD'
18      WRITE(6,6002)T3
6002    FORMAT('T3= ',F9.4)

        TMIN=T1
        DO 702 I=1,9000
        TPRME(I)=T1+(REAL(I))/100.
        IF(TPRME(I).GE.T3)GO TO 800
        TMAX=MAX(TMAX,TPRME(I))
        IMAX=MAX(IMAX,I)

        IF((T1.LT.TPRME(I)).AND.(TPRME(I).LT.T2))GO TO 60
        IF((T2.LT.TPRME(I)).AND.(TPRME(I).LT.T3))GO TO 70

C***** CALCULATION OF LIMITS OF INTEGRATION (T1<T'<T2)
60      A1=U1+(V*TPRME(I))
        A2=U2+(V*TPRME(I))
        D=((BPRME**2.)*Z)-A1
        DD=((BPRME**2.)*Z)-A2
        E1=((Z-A1)**2.)-S1
        E2=((Z-A2)**2.)-S1

        IF(E1.LT.0.)GO TO 200
        E=BPRME*SQRT(E1)
        ZPM=(D-E)/Q
        ZPF=(D+E)/Q
        IF(ZPM.LE.0.)GO TO 61
        IF(ZPM.GT.0.)GO TO 62
61      ZPM=0.
62      ZPI=ZPM
63      IF(ZPF.GE.L)GO TO 64
        IF(ZPF.GT.L)GO TO 110
64      ZPF=L

C***** CALCULATION OF THE FIELD
110     WPI=Z-ZPI
        WPF=Z-ZPF
        W1=WPI/S
        W2=WPF/S
        YY=ATAN(W1)

```



```

XX=ATAN(W2)
B(I)=(ROE*N*(BETA**2.))*(YY-XX)
BMAX=MAX(BMAX,B(I))
GO TO 702

```

C\*\*\*\*\* CALCULATION OF LIMITS OF INTEGRATION (T2<T'<T3)

C\*\*\*\*\* FIRST INTEGRAL

```

70  A1=U1+(V*TPRME(I))
    A2=U2+(V*TPRME(I))
    D=((BPRME**2.)*Z)-A1
    DD=((BPRME**2.)*Z)-A2
    E1=((Z-A1**2.))-S1
    E2=((Z-A2**2.))-S1

    IF(E1.LT.0.)GO TO 200
    E=BPRME*SQRT(E1)
    IF(E2.LT.0.)GO TO 220
    EE=BPRME*SQRT(E2)
    ZPM=(D-E)/Q
    ZPM2=(DD-EE)/Q
    IF(ZPM2.LE.0.)GO TO 73
    IF(ZPM2.GT.0.)TO TO 74
73  ZPF=0.
    GO TO 75
74  ZPF=ZPM2
75  IF(ZPM.LE.0)GO TO 71
    IF(ZPM.GT.0)GO TO 72
71  ZPI=0.
    GO TO 80
72  ZPI=ZPM

```

C\*\*\*\*\* CALCULATION OF THE FIELD FROM THE FIRST INTEGRAL

```

80  WPI=Z-ZPI
    WPF=Z-ZPF
    W1=WPI/S
    W2=WPF/S
    YY=ATAN(W1)
    XX=ATAN(W2)
    B1=(ROE*N*(BETA**2.))*(YY-XX)

```

C\*\*\*\*\* SECOND INTEGRAL

```

    IF(E1.LT.0.)GO TO 200
    E=BPRME*SQRT(E1)
    IF(E2.LT.0.)GO TO 220
    EE=BPRME*SQRT(E2)
    ZPI=(DD+EE)/Q
    ZPF=(D+E)/Q

```

```

      IF(ZPI.GE.L)GO TO 83
      GO TO 84
83      ZPI=L
84      IF(ZPF.GE.L)GO TO 82
      IF(ZPF.LT.L)GO TO 81
82      ZPF=L

C***** CALCULATION OF THE FIELD FROM THE SECOND INTEGRAL
81      WPI=Z-ZPI
      WPF=Z-ZPF
      W1=WPI/S
      W2=WPF/S
      YY=ATAN(W1)
      XX=ATAN(W2)
      B2=(ROE*N*(BETA**2.))*(YY-XX)

C***** TOTAL FIELD
      B(I)=B1+B2

      BMAX=MAX(BMAX,B(I))
702     CONTINUE
      GO TO 800

C***** PATH TO THE LEFT
20      WRITE(6,2052)
2052     FORMAT('PATH TO THE LEFT')
      PATH='LEFT'
      TMIN=TC

304     DO 700 I=1,9000
      TPRME(I)=(TC+REAL(I))/100.
      IF TPRME(I).GE.TB)GO TO 800
      TMAX=MAX(TMAX,TPRME(I))
      IMAX=MAX(IMAX,I)

      A1=U1+(V*TPRME(I))
      A2=U2+(V*TPRME(I))
      D=((BRPME**2.)*Z)-A1
      DD=((BPRME**2.)*Z)-A2
      E1=(Z-A1)**2.-S1
      E2=(Z-A2)**2.-S1

      IF(TA.LT.TD)GO TO 40
      IF(TA.GT.TD)GO TO 45
      IF(TA.EQ.TD)GO TO 50

C***** TA < TD: CALCULATION OF LIMITS OF INTEGRATION
40      TIME='TA < TD'
      IF((TC.LT.TPRME(I)).AND.(TPRME(I).LT.TA))GO TO 41
      IF((TA.LT.TPRME(I)).AND.(TPRME(I).LT.TD))GO TO 42
      IF((TA.LT.TPRME(I)).AND.(TPRME(I).LT.TB))GO TO 43

```



```

C***** TC<T'<TA
41      IF(E1.LT.0.)GOTO 200
        E=BPRME*SQRT(E1)
        ZPM=(D-E)/Q
        ZPF=L
        ZPI=ZPM
        IF(ZPI.LT.0.)GO TO 304
        GO TO 100

C***** TA<T'<TD
42      ZPI=0.
        ZPF=L
        GO TO 100
C***** TA<T'<TB
43      ZPI=0.
        IF(E2.LT.0.)GO TO 220
        EE=BPRME*SQRT(E2)
        ZPM2=(DD-EE)/Q
        ZPF=ZPM2
        IF(ZPF.LT.0.)GO TO 304
        GO TO 100

C***** TA > TD:  CALCULATION OF LIMITS OF INTEGRATION
45      TIME='TA > TD'
        IF((TC.LT.TPRME(I)).AND.(TPRME(I).LT.TD))GO TO 46
        IF((TD.LT.TPRME(I)).AND.(TPRME(I).LT.TA))GO TO 47
        IF((TA.LT.TPRME(I)).AND.(TPRME(I).LT.TB))GO TO 48

C***** TC<T'<TD
46      IF(E1.LT.0.)GO TO 200
        E=BPRME*SQRT(E1)
        ZPM=(D-E)/Q
        ZPF=L
        ZPI=ZPM
        IF(ZPI.LT.0.)GO TO 304
        GO TO 100

C***** TD<T'<TA
47      IF(E1.LT.0.)GO TO 200
        E=BPRME*SQRT(E1)
        IF(E2.LT.0.)GO TO 220
        EE=BPRME*SQRT(E2)
        ZPM=(D-E)/Q
        ZPM2=(DD-EE)/Q
        ZPI=ZPM
        ZPF=ZPM2
        IF(ZPI.LT.0.)GO TO 304
        IF(ZPF.LT.0.)GO TO 304
        GO TO 100

```

```

C***** TA<T'<TB
48      ZPI=0.
        IF(E2.LT.0.)GO TO 220
        EE=BPRME*SQRT(E2)
        ZPM2=(DD-EE)/Q
        ZPF=ZPM2
        IF(ZPF.LT.0.)GO TO 304
        GO TO 100

C***** TA = TD:  CALCULATION OF LIMITS OF INTEGRATION
50      TIME='TA = TD'
        IF((TC.LT.TPRME(I)).AND.(TPRME(I).LT.TA))GO TO 51
        IF((TD.LT.TPRME(I)).AND.(TPRME(I).LT.TB))GO TO 52

C***** TC<T'<TA
51      IF(E1.LT.0.)GO TO 200
        E=BPRME*SQRT(E1)
        ZPM=(D-E)/Q
        ZPF=L
        ZPI=ZPM
        IF(ZPI.LT.0.)GO TO 304
        GO TO 100

C***** TD<T'<TB
52      ZPI=0.
        IF(E2.LT.0.)GO TO 220
        EE=BPRME*SQRT(E2)
        ZPM2=(DD-EE)/Q
        ZPF=ZPM2
        IF(ZPF.LT.0.)GO TO 304
        GO TO 100

C***** CALCULATION OF THE FIELD
100     WPI=Z-ZPI
        WPF=Z-ZPF
        W1=WPI/S
        W2=WPF/S
        YY=ATAN(W1)
        XX=ATAN(W2)
        B(I)=(ROE*N*(BETA**2.))*(YY-XX)
        BMAX=MAX(BMAX,B(I))
700     CONTINUE

200     WRITE(6,201)
201     FORMAT('VALUE OF E1 IS NEGATIVE.  PROGRAM WILL BEGIN
              AGAIN.')
```

GO TO 300

```

220     WRITE(6,221)
221     FORMAT('VALUE OF E2 IS NEGATIVE.  PROGRAM WILL BEGIN
              AGAIN.')
```

GO TO 300

```

C***** BEGIN GRAPHICS *****
C***** INPUT SCALING VALUES *****
800      WRITE(6,1510)BMAX
1510     FORMAT(/'THE MAXIMUM VALUE OF B IS      ',F16.8)
        WRITE(6,1520)
1520     FORMAT(/'ENTER THE MAXIMUM HEIGHT ON THE B AXIS'. $)
        READ(5,*)YMAX
        WRITE(6,1530)
1530     FORMAT(/'ENTER THE B AXIS MARKING INCREMENT  ', $)
        READ(5,*)SDYY
        CALL INETYPE(IPAT)
        CALL COLORLIN(ICOLOR)

        WRITE(6,1560)
1560     FORMAT('DO YOU WANT INFORMATION PRINTED ALONGSIDE
              THE GRAPH?      ', $)
        READ(5,1570)AXCH
A570     FORMAT(A1)

        CALL INSTR1
        PAUSE'#1'

        XMAX=TMAX
        XMIN=TMIN
        SDXX=(TMAX-TMIN)/3.0
        SCALEX=60.0/(XMAX-XMIN)
        SDX=SDXX*SCALEX
        XN=(XMAX-XMIN)/SDXX
        JXMAX=INT(XN)

        YMIN=0.0
        SCALEY=80.0/(YMAX-YMIN)
        SDY=SCALEY*SDYY
        YN=(YMAX-YMIN)/SDYY
        JYMAX=INT(YN)

C***** BEGIN TO PLOT *****
        CALL GRSTRT(4105,1)
        CALL NEWPAG
        CALL VAXES
        CALL VXMARK(JXMAX,SDX)
        CALL VYMARK(JYMAX,SDY)

C***** PLOT GRAPH *****
        CALL MOVE(18.0,19.0)
        CALL DASHPT(IPAT)
        CALL LINCLR

        DO 540 I=1,IMAX
C***** SCALING OF VALUES *****
        X=18.0+60.0*((TPRME(I)-TMIN)*100.)/IMAX
        Y=19.0+80.0*B(I)/YMAX

```

```

CALL DRAW(X,Y)
540    CONTINUE
C***** DECISION TO LABEL GRAPH *****
CALL GRSTOP
CALL INSTR2
PAUSE'#3'
CALL LINE
CALL INSTR1
PAUSE'#4'
CALL GRSTRT(4105,1)

CALL XLABEL(JXMAX,SDX,SDXX,XMIN)
CALL YLABEL(JYMAX,SDY,SDYY,YMIN)
C***** AXES LABELS AND PARAMETER LEGEND *****
IF(AXCH.EQ.'Y')GO TO 556
GO TO 651
556    CALL MOVE(50.0,10.0)
CALL TXICUR(8)
CALL TEXT(11,'TIME (NSEC)')
CALL MOVE(5.0,83.0)
CALL TXICUR(3)
CALL TEXT(1,'B')
REL=REAL(L)
RES=REAL(S)
REZ=REAL(Z)
CALL MOVE(85.0,95.0)
CALL TXICUR(1)
CALL TXFCUR(2)
CALL TEXT(15,'BEAM LENGTH = ')
CALL RNUMBR(REL,1,8)
CALL MOVE(85.0,85.0)
CALL TEXT(15,'                Z = ')
CALL RNUMBR(REZ,1,8)
CALL MOVE(85.0,75.0)
CALL TEXT(15,'                S = ')
CALL RNUMBR(RES,1,8)
CALL MOVE(85.0,65.0)
CALL TEXT(6,PATH)
CALL MOVE(85.0,55.0)
CALL TEXT(8,TIME)

651    CALL GRSTOP
CALL INSTR2
PAUSE'#5'
GO TO 440

C***** DECISION TO PRINTOUT, RE-RUN, PLOT VALUES OR EXIT
440    WRITE(6,445)
445    FORMAT(// ' 1: PRINTOUT VALUES' / ' 2: RUN PROGRAM
              AGAIN' )

```

```

        WRITE(6,450)
450      FORMAT('3:      PLOT  VALUES'/'4:      EXIT'//'ENTER
              CHOICE', $)
        READ(5,460)PRE
460      FORMAT(A1)
        IF(PRE.EQ.'4')GO TO 301
        IF(PRE.EQ.'3')GO TO 800
        IF(PRE.EQ.'2')GO TO 300
        IF(PRE.EQ.'1')GO TO 660
        GO TO 440

C***** PRINT OUT VALUES *****
660      WRITE(6,670)
670      FORMAT('      TIME                      B'/28(' - ')/)
        DO 690 I=1,IMAX
        WRITE(6,680)TPRME(I),B(I)
680      FORMAT(F16.8,2X,F10.8)
690      CONTINUE
        GO TO 440

301      END

```

## LIST OF REFERENCES

1. Jelley, J.V., Cerenkov Radiation and Its Applications, Pergamon Press, 1958.
2. Buskirk, Fred R. and Neighbours, John R., "Time Development of Cerenkov Radiation", Physical Review A, v. 31, Number 6, June 1985.

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time dependent B field  
over a finite path.

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